

**EXAM 6 – CANADA, SPRING 2016**

13. (7.75 points)

The following information is available for a property and casualty insurance company that only writes auto insurance as at December 31, 2015. All amounts are in thousands of dollars (\$000s).

The cumulative accident year payment pattern is as follows:

<b>Age (months)</b>	<b>% cumulative paid</b>
12	25.0%
24	50.0%
36	75.0%
48	100.0%

The following additional information is also available:

- Assume all claim payments are made in the middle of the year.
- The company started writing business on January 1, 2015.
- The exposures, 12 months policies, are uniformly written throughout the year.
- There is no reinsurance.
- Annual Gross Written Premium in 2015 = \$10,000
- Net undiscounted unpaid claims at the end of 2015 = \$5,250
- Margin for Adverse Deviation (MfAD) Claims development: 10.0%
- MfAD Investment return rates: 0.5%
- The budgeted loss ratio for 2016 is equal to 65.0%.
- Maintenance expenses are equal to 3.0% of gross written premiums.
- Assume that maintenance expenses are paid during the time the unearned premium is earned.
- Interest rate shock factor = 1.25%.

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The composition of the company's bond portfolio is as follows:

<b>Description</b>	<b>Bond #1</b>	<b>Bond #2</b>
Maturity Date	31/12/2016	31/12/2017
Annual Coupon Rate	2.60%	3.50%
# of coupons per year	2	2
Par value	\$10,000	\$10,000
Coupons value (\$)	\$130	\$175
Annual effective yield	2.31%	3.23%
Modified Duration	0.972	1.888
Market Value	\$10,030	\$10,060

Note that there are no other invested assets other than Bond #1 and Bond #2 and that the combined market yield of the investment portfolio is used to discount losses.

Calculate the margin required for interest rate risk for the current year.

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**SAMPLE ANSWERS AND EXAMINER'S REPORT**

**QUESTION 13**

**TOTAL POINT VALUE: 7.75**

**LEARNING OBJECTIVE: C1 and C2**

**SAMPLE ANSWERS**

**Part a: 7.75 points**

1) Calculation of the duration of the bond portfolio

(1)	(2)	(3)	(4)
Asset	Market value	Modified duration	Yield
Bond #1	10,030	0.972	2.31%
Bond #2	10,060	1.888	3.23%
<b>Total</b>	<b>20,090</b>	<b>1.431</b>	<b>2.92%</b>

Portfolio duration =  $(10,030 \times 0.972 + 10,060 \times 1.888) / (20,090) = 1.431$  (weighed by market value)

Portfolio yield =  $(10,030 \times 0.972 \times 2.31\% + 10,060 \times 1.888 \times 3.23\%) / (20,090 \times 1.431) = 2.92\%$  (weighed by market value and modified duration)

2) Present Value of unpaid losses

**Adjusted payout pattern**

(1)	(2)	(3)	(4)
Timing	Cumulative Payout	Incremental Payout	Unpaid Payout
0.5	25.0%	25.0%	33.3%
1.5	50.0%	25.0%	33.3%
2.5	75.0%	25.0%	33.3%
3.5	100.0%	25.0%	
4.5	100.0%		

Timing 0.5:  $25\% / 75\% = 33.3\%$

Timing 1.5:  $25\% / 75\% = 33.3\%$

Timing 2.5:  $25\% / 75\% = 33.3\%$

**SAMPLE ANSWERS AND EXAMINER'S REPORT**

**Present Value Unpaid Losses**

(1)	(2)	(3)	(4)	(5)	(6)
Timing	Payout	PV Factor @ 2.92%	PV @ 2.92%	PV Factor @ 2.42%	PV @ 2.42%
0.5	1,750	0.986	1,725.50	0.988	1,729.00
1.5	1,750	0.958	1,676.50	0.965	1,688.75
2.5	1,750	0.931	1,629.25	0.942	1,648.50
<b>Total</b>	<b>5,250</b>		<b>5,031.25</b>		<b>5,066.25</b>

Macaulay Duration      **1.481**

Modified Duration      **1.439**

(2) = Unpaid Payout \* \$5,250

(3) =  $(1 + 2.92\%)^{-t} - (1)$

(4) = (2) \* (3)

(5) =  $(1 + 2.92\% - 0.50\%)^{-t} - (1)$

(6) = (2) \* (5)

Macaulay Duration =  $\frac{\text{sumproduct}((4);(1))}{(4)_{\text{total}}}$

Modified duration = Macaulay duration /  $(1 + 2.92\%)$

Net Unpaid Undiscounted = \$5,250.00

Net Unpaid Discounted = \$5,031.25

PfAD claims development = \$503.13 = 5031.25 \* 10%

PfAD Interest Rate = \$35.00 = \$5066.25 - \$5031.25

Net Unpaid Disc. w/ PfAD = \$5569.38

**SAMPLE ANSWERS AND EXAMINER'S REPORT**

3) Premium liabilities

**Premium Liabilities Duration**

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Timing	Cumulative Payment Pattern (Interpolated)	In Year Payment	PV Factor @ 2.92%	PV @ 2.92%	PV Factor @ 2.42%	PV @ 2.42%	Age at EOY
0.2929	30.18%	30.18%	0.992	0.299	0.993	0.300	0.7071
1.2929	55.18%	25.00%	0.963	0.241	0.970	0.243	1.7071
2.2929	80.18%	25.00%	0.936	0.234	0.947	0.237	2.7071
3.2929	100.00%	19.82%	0.91	0.18	0.924	0.183	3.7071
<b>Total</b>				<b>0.954</b>		<b>0.963</b>	

Macaulay Duration **1.602**

Modified Duration **1.557**

(1) To adjust average payment date for UPR exposure, assume x to be the time to end of the year from the average payment of the UPR. The average payment is the time that would split the UPR triangle in half. The area of the triangle is 72 (12 \* 12 / 2). To solve for x,  $x^2/2 = 36$ . Thus  $x = 8.485$  months, which is 0.7071 years. So from the beginning of the year the average payment is at 1-x or 0.2929 years. Also, 1/3 should be accepted since it represents the average accident date for premium liabilities.

(2) Claims will occur on average 0.2929 years after the December 31 valuation date. At the end of the first calendar year, claims in connection with unearned premium will be 1.0000 - 0.2929 = 0.7071 years old on average. The cumulative payment pattern for these claims is therefore interpolated between a cohort of claims that are 0.5 years old (assumed payment pattern at 12 months) and 1.5 years old (assumed payment pattern at 24 months). The cumulative payment pattern is linearly interpolated as follows:

$$[(0.7171 - 0.5)/(1.5 - 0.5)] \times (50\% - 25\%) + 25\%$$

The linear interpolation is similar in subsequent years.

$$(4) = (1 + 2.92\%)^x - (1)$$

$$(5) = (3) * (4)$$

$$(6) = (1 + 2.92\% - 0.50\%)^x - (1)$$

$$(7) = (3) * (6)$$

$$\text{Macaulay Duration} = \text{sumproduct}((5);(1)) / (5)_{\text{total}}$$

$$\text{Modified duration} = \text{Macaulay duration} / (1 + 2.92\%)$$

**SAMPLE ANSWERS AND EXAMINER'S REPORT**

**Maintenance expenses:**

Undiscounted =  $10,000 * 3.0\% = 300$  (using unearned premium to calculate maintenance expense is also accepted  $(5,000 * 3.0\%)$ )

Discounted =  $300 * (1.0292)^{-0.2929} = 297.60$

Macaulay Duration =  $297.60 * 0.2929 / 297.60 = 0.2929$

Modified Duration =  $0.2929 / (1.0292) = 0.285$

**Premium Liabilities:**

Unearned Premium = 5,000.00

Loss ALAE =  $5,000 * 65\% = 3,250.00$

PV Loss ALAE =  $3,250 * 0.954 = 3,100.50$

PfAD claims development =  $3,100.50 * 10\% = 310.05$

PfAD Interest Rate =  $3,250 * (0.963 - 0.954) = 29.25$

PV with PfAD = 3,439.80

Policy Liabilities in connection with Unearned Premium =  $3,439.80 + 300 = 3,739.80$

Total Duration Premium Liabilities =  $(3439.80 * 1.557 + 297.6 * 0.285) / (297.6 + 3439.80) = 1.456$

4) Interest rate risk margin

	(1)	(2)	(3)	(4)
	Fair Value	Modified Duration	Positive Shock +1.25%	Negative Shock (1.25%)
<b>Interest Sensitive Assets</b>				
Bond portfolio	20,090	1.431	359.36	(359.36)
<b>Total</b>			<b>359.36</b>	<b>(359.36)</b>
<b>Interest Sensitive Liabilities</b>				
Net unpaid claims and adjustment expenses	5,569	1.439	100.17	(100.17)
Net premium liabilities	3,740	1.456	68.06	(68.06)
<b>Total</b>			<b>168.23</b>	<b>(168.23)</b>

(3) = (1)\*(2)\*1.25%

(4) = (1)\*(2)\*(-1.25%)

## SAMPLE ANSWERS AND EXAMINER'S REPORT

Capital required for  $\Delta y$  shock increase =  $\text{MAX}(0; 359.36 - 168.23) = 191.13$

Capital required for  $\Delta y$  shock decrease =  $\text{MAX}(0; -359.36 - -168.23) = 0$

Total interest rate risk margin =  $\text{MAX}(191.13 ; 0) = 191.13$

### EXAMINER'S REPORT

- This question combined MCT and premium liability.
- The candidates were expected to know how to evaluate policy liabilities in accordance with accepted practice in Canada and to calculate interest rate risk margin component of the MCT.
- It is understood that this is a long and tough question, even for the well-prepared candidates as the question is not broken down to subparts to guide through the calculation process.
- Most candidates managed to obtain partial credit. Most difficult parts were to calculate the premium liabilities and the durations.
- Few candidates used another method to calculate interest rate risk margin. They calculated the value of liabilities with a positive shock and the negative shock. They used those values and compared them the APV of liabilities. They did the same thing for the asset and compare them with market value of the bonds. This solution was accepted but no candidates got full credit.
- Some candidates used effective duration. The answer has been accepted.
- Some candidate used unearned premium to calculate maintenance expense instead of written premium as asked in the question. Both answers were accepted.
- Most of the candidates are having problem in the calculation of duration, particularly the maintenance expense duration.