## IRS Loss Reserve Discounting

## Statutory vs tax-basis loss reserves

Underwriting income is premium revenue minus underwriting + acquisition expenses and incurred losses.

- Statutory calendar year incurred losses = paid losses plus the change in full value loss reserves from the beginning to the end of the year.
- Tax basis incurred losses = paid losses plus the change in the discounted loss reserves from the beginning to the end of the year.

Tax basis incurred losses include loss adjustment expenses, both allocated (defense and cost containment) and unallocated (adjusting and other). ${ }^{1}$

Total full value incurred losses equal total discounted incurred losses over the life of the claims. A claim estimated to be paid for $L$ in $N$ years is a discounted loss of $\mathrm{L} /(1+r)^{\mathrm{N}}$ when it occurs. Each subsequent year the discounted loss includes the unwinding of the interest discount $=\mathrm{L} /(1+\mathrm{r})^{\mathrm{t}-1}-\mathrm{L} /(1+\mathrm{r})^{\mathrm{t}}$. The total discounted loss is the total undiscounted loss:

$$
\frac{L}{(1+r)^{N}}+\sum_{t=1}^{N}\left(\frac{L}{(1+r)^{t-1}}-\frac{L}{(1+r)^{t}}\right)=L
$$

Loss reserve discounting affects the pattern of income recognition. Statutory accounting recognizes the losses more quickly than tax accounting does, so taxable income is recognized more quickly than statutory income. Expected underwriting income for long-tailed lines of business differs for statutory vs taxable income:

- Statutory accounting shows underwriting losses when accidents occur and investment income as claims settle. Insurers hold additional capital supporting the full value loss reserves so that claims are paid even in adverse scenarios.
- In tax accounting, the investment income on the assets supporting loss reserves offsets the accrual of the interest discount on the losses. The insurer shows an underwriting gain during the policy term and no subsequent gains or losses, except for investment income on capital and surplus.

Illustration: An insurer writes medical malpractice policies with an expense ratio of $30 \%$ and a discounted loss ratio of $65 \%$, leaving a $5 \%$ profit margin. Losses are paid five years after they occur, and the five-year risk-free interest rate is $7 \%$ per annum. The ultimate loss ratio is $65 \% \times 1.07^{5.5}=94.30 \%$ : half a year between premium collection and loss occurrence + five years from loss occurrence until payment. The statutory underwriting margin of $1-30 \%-94.3 \%=-24.30 \%$ is offset by investment income in later years. If the tax rate is zero, the economic profit margin is still $5 \%$.

Economic income during the policy term is the present value of future profits, or the net premium minus the present value of the loss payments. By using discounted losses as the offset to taxable income, the IRS taxes the economic profit margin, not the underwriting profit margin. As the losses settle, the investment income on the assets backing the loss reserves offsets the amortization of the interest discount in the reserves. The full gain or loss is realized during the policy term, with no expected gain or loss in subsequent years. ${ }^{2}$

Illustration: A policy written on January 1, 20X4, for a premium of $\$ 10,000$ has one loss on 12/31/20X4 that is paid for $\$ 13,310$ on $12 / 31 / 20 \times 6$. The term structure of interest rates is flat at $10 \%$ per annum. Assume the IRS loss payment pattern is the same as the actual payment pattern and the IRS discount rate is also $10 \%$.

Statutory accounting shows an underwriting loss of $\$ 10,000-\$ 13,310=\$ 3,310$ in $20 \times 4$ and zero in 20X5 and 20X6. The investment income is $\$ 10,000 \times 10 \%=\$ 1,000$ in $20 X 4, \$ 11,000 \times 10 \%=\$ 1,100$ in 20X5, and $\$ 12,100 \times 10 \%=\$ 1,210$ in 20X6; the total investment income is $\$ 3,310$.

If we assume a two year IRS loss payment pattern and a discount rate of $10 \%$ per annum, the discounted reserves are $\$ 13,310 / 1.100^{2}=\$ 11,000$ at $12 / 31 / 20 X 4$. The tax basis underwriting income of $-\$ 1,000$ offsets the $\$ 1,000$ of investment income, and the tax liability is zero.

In 20X5, investment income is $\$ 11,000 \times 10 \%=\$ 1,100$. The discounted loss reserve on $12 / 31 / 20 X 5$, is $\$ 13,310 / 1.100=\$ 12,100$. The underwriting loss (or the offset to underwriting income) for tax year 20X5 is the amortization of the interest discount on the reserves, or $\$ 12,100-\$ 11,000=\$ 1,100$. The underwriting loss offsets the investment income, and the tax liability is zero.

In 20X6, investment income is $\$ 12,100 \times 10 \%=\$ 1,210$. The incurred loss offset to taxable underwriting income in $20 \times 6$ is the paid loss plus the change in the discounted loss reserve, or $\$ 13,310$ (paid on $12 / 31 / 20 \times 6)+\$ 0-\$ 12,100=-\$ 1,210$. This is the amortization of the interest discount on the 12/31/20X5 reserve of $\$ 11,000$. It offsets the investment income in 20X6, and the tax liability is zero.

If incurred losses by calendar year are not increasing or decreasing, expected statutory incurred losses equal expected tax basis incurred losses, since the higher statutory incurred loss at the accident date equals the accrual of interest for losses from previous accident years.

General accounting rules do not affect present values, which depend on cash flows, not financial statements. But tax accounting rules affect the tax cash flows and the after-tax present value of underwriting operations.

Before the Tax Reform Act of 1986, the tax basis underwriting income equaled statutory underwriting income, reducing tax basis underwriting income and the associated tax liability during the policy year and increasing tax basis underwriting income in subsequent years. In effect, the IRS gave insurers interest free loans equal to the tax rate times the present value of the interest discount in the full value loss reserves. The loans were amortized over the life of the claims, and insurers earned the after-tax investment income on these loans. ${ }^{3}$ After 1986, the slower recognition of incurred losses in tax accounting raises underwriting income and the associated tax liability when losses occur and lowers underwriting income and the associated tax liability as loss reserves run off. Insurers pay tax on economic underwriting income, not statutory underwriting income.

Illustration (continued from above): The interest discount in the reserves is $\$ 13,310 \times\left(1-1 / 1.10^{3}\right)=\$ 3,310$, with a present value of $\$ 3,000$. With a tax rate of $\tau$, before 1986 the IRS gave insurers an interest-free loan of $\$ 3,000$ when the policy was written, to be amortized as the claims settle. Insurers received the investment income on this loan. ${ }^{4}$

Tax accounting for insurers: U.S. vs other countries
In the United States, taxation of insurers depends on the NAIC Annual Statement. In 1986, neither GAAP nor statutory accounting showed discounted reserves, so the IRS developed its own computation method.

In most other countries, taxation of insurers uses general accounting statements (GAAP or IFRS). Accounting for insurers now uses fair value loss reserves as defined in IFRS 4, "Insurance contracts," with a revised draft to be issued in 2018 (and re-named IFRS 17). ${ }^{5}$

Illustration: Before IFRS 4, general accounting systems in most countries used full value (undiscounted) loss reserves as the offset to underwriting income, and tax accounting deducted the full value loss reserves from premium revenue to derive taxable income. A few countries had tax basis loss reserve discounting:

Germany used a $5.5 \%$ discount rate for reserves with remaining lifetimes of one year or more.

- Austria considered $70 \%$ of loss reserves as short-term and fully tax deductible and $30 \%$ of reserves as long-term and $80 \%$ tax deductible: $70 \%+30 \% \times 80 \%=94 \%$, so $6 \%$ of reserves are not tax deductible

IFRS 4 reserve values are intended to reflect market values. ${ }^{6}$ Tax systems in other countries will presumably use IFRS 17 fair value (discounted) loss reserves once it is implemented. ${ }^{7}$

Tax basis reserves use undiscounted reserves, discount rates, and payment patterns.
The tax basis discounted loss reserves are determined from (i) undiscounted loss reserves, (ii) discount rates promulgated each year by the Treasury, and (iii) loss payment patterns by line of business. The illustration shows the concepts, though the details differ from the IRS computation; the IRS procedure is further below.

Illustration: The 12/31/20X4 undiscounted loss reserves are $\$ 100$ million. The loss reserve discount rate is $8 \%$ per annum. The $\$ 100$ million of reserves will be paid $50 \%$ on $12 / 31 / 20 \times 5,30 \%$ on $12 / 31 / 20 \times 6$, and $20 \%$ on $12 / 31 / 20 \times 7$. $^{8}$ The discounted loss reserves are

$$
\$ 100 \text { million } \times\left(50 \% / 1.08+30 \% / 1.08^{2}+20 \% / 1.08^{3}\right)=\$ 100 \text { million } \times 0.879=\$ 87.9 \text { million. }
$$

## .....Undiscounted loss reserves are from Schedule P.

The loss reserves on the statutory balance sheet (Underwriting and Investment Exhibit) might be discounted; the loss reserves in Schedule P, Part 1, are undiscounted values. If the loss reserves on the balance sheet or in Schedule P are discounted and the discount is disclosed on (or with) the Annual Statement, the losses may be grossed up before the IRS loss reserve discounts are applied. ${ }^{9}$

Schedule P, Part 1, reserves are gross of non-tabular discount and net of tabular discount, so the grossing-up is normally for tabular discounts. If the Schedule $P$ workers' compensation reserves for a given accident year are $\$ 10$ million, including $\$ 1$ million of tabular discount, and the applicable IRS discount factor is $85 \%$, the discounted reserves are $(\$ 10$ million $+\$ 1$ million $) \times 85 \%=\$ 9.35$ million.

An insurer might inflate its nominal reserves and claim an excessive loss reserve discount, leaving its statutory reserves the same but reducing the tax liability. To prevent this, the discounted IRS loss reserves by line of business and accident year may not be greater than the loss reserves in the Annual Statement. ${ }^{10}$

The statutory loss reserves are generally greater than the IRS loss reserves. ${ }^{11}$ The workers' compensation prior years row (Part 1D) is an exception. These reserves are primarily indemnity reserves for lifetime pension cases, and many companies use tabular discounts. For this row, the composite discount factor in the IRS discounting calculations assumes (on average) three more years of payment, whereas the pension cases in these reserves may have future expected lifetimes of 10 to 20 years. ${ }^{12}$

Illustration: Suppose the workers' compensation prior years row shows unpaid losses and LAE of $\$ 30$ million. In the Notes to the Financial Statements, the company reports a $\$ 10$ million tabular discount for these claims. The IRS composite discount factor for these reserves is $90 \%$. Without the limit, the gross loss reserves are $\$ 30$ million $+\$ 10$ million $=\$ 40$ million. The IRS discounted loss reserves are $90 \% \times \$ 40$ million $=\$ 36$ million. This exceeds the $\$ 30$ million statutory loss reserves, so the tax basis reserves are capped at $\$ 30$ million.

## Market consistent discount rates vary by calendar year.

The IASB and the FASB now show most assets and liabilities at fair value, or the "price that would be received to sell an asset or paid to transfer a liability in an orderly transaction between market participants at the measurement date" (IFRS 13 for IASB; ASC 820 for FASB). If a market for an asset or liability does not exist,
a valuation method, such as the present value of future cash flows, may be used instead. A market consistent value is one that equals the market value.

Illustration: The present value of future cash flows, using a discount rate based on the financial strength rating of the issuer, is often used to value non-traded bonds, since this valuation method gives market consistent values for publicly traded bonds. ${ }^{13}$

Comparison of market values and cash flows for traded bonds indicates that market consistent discount rates vary by currency and duration, and that market values include an offset for expected default losses. Some economists presume that discount rates also reflect liquidity of the cash flows or volatility of the cash flows, though empirical evidence of the relation is hard to establish. ${ }^{14}$

Discount rates are inferred from current market values, so market values vary with current discount rates, not the discount rates when the asset was acquired or the liability was incurred. ${ }^{15}$ If interest rates rise one percentage point, the market value of a bond with an effective duration of $N$ years declines about $N \%$. ${ }^{16}$

IFRS 4 values loss reserves at the current fair value, so it uses current interest rates, not averages of past interest rates. Market values for traded assets use current rates, so market consistent valuation of assets and liabilities uses current discount rates for non-traded assets as well, such as loss reserves.

For most assets, IFRS 9 (investment accounting) requires that the market value effect of a change in interest rates be included in current financial statements (either profit \& loss accounts or other comprehensive income, depending on the type of asset). ${ }^{17}$ Similarly, IFRS 4 requires that the fair value effect on loss reserves of a change in interest rates be included in other comprehensive income. ${ }^{18}$

IRS discount rates vary by accident year, not calendar year.
The IRS loss reserve discounting assumes that insurers buy investment grade Treasury securities when they collect the policy premium. NAIC statutory accounting shows investment grade bonds at amortized value: the implicit discount rate for the bond cash flows is the rate when the bond is purchased, not the current rate.

Tax basis values of insurers' bonds are the statutory values, so changes in market rates do not change the bond's book value. To match the unwinding of the interest discount on the loss reserves with the associated investment income on the supporting assets, the loss reserve discount rate is fixed during the policy year and applies to losses in the accident year until they settle. The loss reserve discount rate should use the best estimate of interest rates over the life of the claims.

Most financial economists presume that interest rate paths have little autocorrelation: the change from today's rate to tomorrow's rate is not materially correlated with the change from yesterday's rate to today's rate. The discount rate for fair values should use the best estimate of future rates. If the autocorrelation is zero and the interest rate process is a random walk, the current rate is the best predictor of future rates. If interest rates vary stochasticity (random deviations) with reversion to the mean, an average rate may be a better predictor of future rates than the current rate is. Most interest rate models assume weak (if any) mean reversion (such as $4 \%$ or $5 \%$ per annum) but significant stochasticity, so the average should be heavily weighted toward the most recent periods. ${ }^{19}$

For each accident year, the discount rate is the 60 month moving average of the federal mid-term rates ending on the December 1 preceding the accident year. This rate is vintaged (frozen) and applies to that accident year's losses in all future calendar years. ${ }^{20}$

The federal mid-term rate is the average rate on Treasury securities with 3 to 9 years remaining maturity, ${ }^{21}$ and is promulgated by the Treasury each month. ${ }^{22}$ The 60 month moving average for an accident year can be determined once the last federal mid-term rate has been announced.

Illustration: The loss reserve discounting rate for accident year 20X9 is the 60 month average of the federal mid-term rates from January 1, 20X4, through December 1, 20X8. It can be computed in mid-December 20X8, before the inception of accident year 20X9.

The average loss date for accident year 20X9 is July 1, 20X9. The average mid-term rate of the 60 month moving average is June 15, 20X6. The average tax basis interest rate is three years earlier than the average loss date. If interest rates have been declining (increasing), tax basis discounted reserves are less (more) than market consistent discounted reserves. ${ }^{23}$

Summary: The IASB approach assumes interest rates follow random walks. The market values of future cash flows are based on the current term structure of interest rates; the date the accident occurs is not relevant.

The IRS rationale is that the insurer uses the premium cash flows to purchase fixed-income securities during the policy year to fund future loss payments. The yield on the fixed-income securities is fixed at the date of purchase. If the duration of the assets backing the reserves matches the duration of the loss liabilities, the losses will be paid from the coupon income and the principal repayment from these securities. The yield during the accident year is the relevant investment yield throughout the life of the policies.

The IRS loss reserve discount factors are by Schedule $P$ line of business, either for the industry or for the insurer. ${ }^{24}$ In contrast, the loss reserve discount factors in IFRS 4 are for the block of business being valued.

Illustration: The average lag between premium collection and loss payment for large dollar deductible business may be many years, and the average lag for first dollar business is only a few years. The IRS loss reserve discount factors are the same for the two sets of business; both use the same Schedule P data.

Loss payment patterns are ratios of paid losses to ultimate losses.
Loss reserve discount factors are ratios of discounted reserves to undiscounted reserves. Actuarial loss payment patterns are often derived from ratios of paid losses to the preceding reserve. Exponential decay, a common payment pattern, assumes paid losses are a fixed percentage of the preceding reserve.

The IRS derives the loss payment pattern from ratios of paid losses in past accident years to ultimate losses for those years. The loss payment pattern and the 60-month moving average discount rate give the discounted losses for these past accident years. The discount factors for future accident years are derived as ratios of these discounted losses to undiscounted losses.

Reserve discounting uses calendar year payment patterns for accident years of reserves. Pricing actuaries often use quarterly patterns; the IRS uses annual periods, since only annual data are in Schedule P, and it assumes payments on July 1 each year, as a proxy for an even distribution of payments during the year. ${ }^{25}$ Schedule $P$ has 10 year loss triangles, so the IRS uses a ten year loss payment pattern plus a five year extension for long-tailed lines of business.

Suppose we derive a loss payment pattern for the accident year 2011 reserves to compute discounted losses. Let $p_{i}$ be the percentage of ultimate losses paid in each future calendar year $i$ : $p_{1}$ is the percentage paid in 2012, $p_{2}$ is the percentage paid in 2013, and so forth. [These percentages differ by line of business and by accident year.] Assuming payments in the middle of each year and a risk-free interest rate $r_{f}$, the discounted loss reserves are the full value losses times $\left\{\left(p_{1} /\left(1+r_{r}\right)^{0.5}+p_{2} /\left(1+r_{r}\right)^{1.5}+p_{3} /\left(1+r_{r}\right)^{2.5}+\ldots\right\}\right.$. The discount factor is the discounted losses divided by the undiscounted losses. (This description is heuristic; the computations are explained more fully below.)

We estimate the percentages $p_{i}$ from past data. ${ }^{26}$ If the payment pattern is stable, the percentage of accident year $X$ losses paid in calendar year $X+k$ is the same as the percentage of accident year $X-j$ losses paid in calendar year $X-j+k$.

Illustration: The percentage of accident year 2011 losses paid in 2012 is the same as the percentage of accident year 2008 losses paid in 2009, as derived from the 2009 Schedule P (available by year end 2010). ${ }^{27}$ Similarly, the percentage of accident year 2011 losses paid in 2013 is the same as the percentage of accident year 2007 losses paid in 2009. The 2009 Schedule $P$ provides nine percentages.

- percentage of 2000 losses paid in 2009 = percentage of 2011 losses paid in 2020.
- percentage of 2001 losses paid in $2009=$ percentage of 2011 losses paid in 2019.
percentage of 2008 losses paid in $2009=$ percentage of 2011 losses paid in 2012.
The IRS derivation of the loss payment pattern is confusing at first. To clarify the procedure, we explain first the standard actuarial derivation of loss payment patterns and then the changes for the IRS procedure.


## Actuarial loss payment patterns

Actuaries derive loss payment patterns from incremental losses paid in each development period divided by either ultimate losses or unpaid losses at the beginning of the period. The Annual Statement does not show triangles of incremental losses, but they may be derived from cumulative paid loss triangles, such as Schedule P, Part 3. The right most column in the 2009 Schedule P, Part 3, triangle shows cumulative losses and defense and cost containment expenses paid by 12/31/2009; the penultimate column shows the corresponding amount paid by $12 / 31 / 2008$. The differences between the two column is the amount paid in 2009. This amount divided by the ultimate losses and defense and cost containment expenses for each accident year is the percentage paid in 2009. Similar computations for other pairs of adjacent columns yield the incremental paid losses in each development period.

Ultimate losses are not known until all claims settle, so they are derived by fitting paid loss percentages to a mathematical distribution. Alternatively, ultimate losses plus defense and cost containment expenses are in the right-most column of Schedule P, Part 2 (and are also shown in Part 1).

Illustration: Assume the paid loss triangle shows the payments by accident year in Exhibit 1. To keep the illustration clear, we show the computations for the most recent development period only.

Exhibit 1: Loss Payment Pattern from Successive Maturities $(\$ 000,000)$
(Data from Schedule P, Parts 2 and 3, from the 20X9 Annual Statement)

| Accident Year <br> (1) | Cum Paid by <br> 20X8 (2) | Cum Paid by <br> 20X9 (3) | Paid in 20X9 <br> (4) | Ultimate <br> Losses (5) | Percentage <br> Paid (6) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $20 X 0$ | $\$ 422$ | $\$ 433$ | $\$ 11$ | $\$ 486$ | $2.26 \%$ |
| $20 X 1$ | $\$ 442$ | $\$ 454$ | $\$ 12$ | $\$ 520$ | $2.31 \%$ |
| $20 X 2$ | $\$ 391$ | $\$ 403$ | $\$ 12$ | $\$ 475$ | $2.53 \%$ |
| $20 X 3$ | $\$ 416$ | $\$ 434$ | $\$ 18$ | $\$ 522$ | $3.45 \%$ |
| $20 X 4$ | $\$ 504$ | $\$ 534$ | $\$ 30$ | $\$ 667$ | $4.50 \%$ |
| $20 X 5$ | $\$ 490$ | $\$ 542$ | $\$ 52$ | $\$ 707$ | $7.36 \%$ |
| $20 X 6$ | $\$ 463$ | $\$ 546$ | $\$ 83$ | $\$ 787$ | $10.55 \%$ |
| $20 X 7$ | $\$ 353$ | $\$ 485$ | $\$ 132$ | $\$ 802$ | $16.46 \%$ |
| $20 X 8$ | $\$ 152$ | $\$ 406$ | $\$ 254$ | $\$ 866$ | $29.33 \%$ |
| $20 X 9$ |  | $\$ 156$ | $\$ 156$ | $\$ 898$ | $17.37 \%$ |

The columns show the following figures:
Column (2): Cumulative dollars of loss paid through 12/31/20X8 (from Part 3),
Column (3): Cumulative dollars of loss paid through 12/31/20X9 (from Part 3).
Column (4): Incremental dollars of loss paid in $20 X 9$ (= column (2) minus column (1)).
Column (5): Incurred losses (from Part 2 or from actuarial estimates).
Column (6): Incremental dollars of loss paid as a percentage of incurred losses (row 3 / row 4).
Consider the row for accident year 20X4:
Column 2: $\$ 504,000$ has been paid by $12 / 31 / 20 \times 8$, or 60 months since inception of the accident year.
Column 3: $\$ 534,000$ has been paid by $12 / 31 / 20 X 9$, or 72 months since inception of the accident year.
Column 4: $\$ 30,000$ was paid between 60 months and 72 months.
Column 5: The total accident year 20X4 incurred losses are \$667,000.
Column 6: $4.5 \%(=\$ 30,000 / \$ 667,000)$ of incurred losses are paid between 60 months and 72 months from inception of the accident year.

In practice, actuaries use all data, not just the latest development period, to smooth random fluctuations in the paid loss percentages. Using weighted averages or ex-high-low averages and graduating the paid loss percentages give more robust and accurate results.

Actuaries already do these calculations for reserve indications. The cumulative paid loss percentage is the reciprocal of the paid loss development factor, so the incremental paid loss percentages are estimated as difference between loss development factors:

$$
\text { paid loss percentage from maturity } Z \text { to maturity } Z+1=1 / L D F_{z+1}-1 / \text { LDF }_{z}
$$

where $L D F_{z}$ is the paid loss development factor from maturity $Z$ to ultimate. Reserving actuaries derive loss development factors by a variety of methods to form reserve indications. The same loss development factors provide the loss payment patterns. For loss reserve discounting, the ratios are unpaid losses, not to incurred losses, so the paid loss percentages are multiplied by the ratio of incurred losses to unpaid losses. ${ }^{28}$

## IRS loss payment patterns

The IRS loss payment pattern is conceptually the same as the actuarial payment pattern, but the IRS derives the pattern from differences between accident years at one valuation date instead of differences between maturities for the same accident year (see Exhibit 2).

- The procedure above uses incremental paid loss percentages (= the difference between cumulative paid loss percentages) by accident year to estimate the percentage of losses paid in each time interval.
- The IRS uses the difference in the cumulative paid loss percentages between successive accident years from Schedule P, Part 1, to estimate the incremental paid loss percentages. ${ }^{29}$

Exhibit 2: Loss Payment Pattern Between Accident Years $(\$ 000,000)$ (Data from Schedule P, Part 1, from the 20X9 Annual Statement)

| AccYr | $20 \times 0$ | $20 X 1$ | $20 X 2$ | $20 X 3$ | $20 \times 4$ | $20 X 5$ | $20 \times 6$ | $20 X 7$ | $20 X 8$ | $20 X 9$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Row 1 | 433 | 454 | 403 | 434 | 534 | 542 | 546 | 485 | 406 | 156 |
| Row 2 | 486 | 520 | 475 | 522 | 667 | 707 | 787 | 802 | 866 | 898 |
| Row 3 | 0.89 | 0.87 | 0.85 | 0.83 | 0.80 | 0.77 | 0.69 | 0.60 | 0.47 | 0.17 |
| Row 4 | 0.02 | 0.02 | 0.02 | 0.03 | 0.03 | 0.07 | 0.09 | 0.136 | 0.295 | 0.174 |

1. Row 1: cumulative paid losses at $12 / 31 / 20 \times 9$ by accident year (Sch P, Part 1, column 11, "total net paid")
2. Row 2: estimated incurred losses at 12/31/20X9 by accident year (Sch P, Part 1, column 28, "total losses and loss expense incurred") ${ }^{30}$
3. Row 3: ratio of cumulative paid losses to incurred losses.
4. Row 4: incremental payments: the incremental paid from 0 to 12 months is the cumulative paid at 12 months (17.4\%). The cumulative paid at 24 months is $46.9 \%$ (accident year 20X8); subtracting the $17.4 \%$ paid by 12 months gives $29.5 \%$ for 12 to 24 months. ${ }^{31}$

We continue in this fashion for the ten accident years in Schedule P, Part 1. If the cumulative paid losses for the oldest year equal the incurred losses, we stop; otherwise, we extend the payment pattern for up to six additional years. The procedure is explained in the examples below: illustration A shows a line of business with no extension of payments and illustration $B$ shows a line with extension of payments.

## Illustration with no extension of payments

An insurer elected to use its own loss payment pattern in the 2007 determination year; this election applies to accident years 2007-2011. We are computing loss reserve discount factors for accident year 2011, using the figures in Exhibit 3 from the insurer's 2009 Schedule P, Part 1.

Exhibit 3: Paid and Incurred Losses from the 2009 Schedule P, Part 1

| Accident | Losses + LAE <br> $\frac{\text { Year }}{\text { Prior }}$ | Paid |
| :---: | :---: | :---: |
| 2000 | 250,000 | $\underline{\text { Incurred }}$ |
| 2001 | 270,000 | 250,000 |
| 2002 | 300,000 | 316,500 |
| 2003 | 320,000 | 348,000 |
| 2004 | 340,000 | 386,500 |
| 2005 | 350,000 | 421,500 |
| 2006 | 370,000 | 480,500 |
| 2007 | 380,000 | 550,500 |
| 2008 | 360,000 | 610,000 |
| 2009 | 330,000 | 687,500 |

The discount rate is the 60 month average of the federal mid-term rates from 1/1/2006 through 12/1/2010. For this illustration, we assume that this average is $7 \%$ per annum.

The computed loss reserve discount factors are for accident year 2011 only. The discount factors for previous accident years at all future valuation dates have already been determined and vintaged; they are not revised.
.....Schedule P gives undiscounted percentages by year.
Exhibit 10 (Illustration A) derives the loss reserve discount factors from Schedule $P$, Part 1, entries.

- Column 2 shows cumulative net paid loss and LAE by accident year at the current statement date.
- Column 3 shows the incurred net losses and LAE by accident year at the current statement date.
- Column 4 shows the cumulative percentage paid from inception of the accident year to the current statement date (column $2 \div$ column 3 ).
- For accident year 2009, the percentage is $\$ 200,000 / \$ 571,500=35 \%$.
- For accident year for 2008, the percentage is $\$ 330,000 / \$ 687,500=48 \%$.
- Column 5, the incremental percentage paid in each year, are first differences of the series in Column 4.
- For accident year 2009, the most recent year, the incremental percent paid is the cumulative percent paid, or $35 \%$.
- For accident year 2008, the cumulative percentage paid at 24 months is $48 \%$, implying that $48 \%-$ $35 \%=13 \%$ of incurred losses are paid 12 months to 24 months from inception of the accident year.

From the ten accident years in Schedule P, we calculate 10 annual intervals of expected loss payments. Any losses unpaid at the end of 10 years are assumed to be paid in the eleventh year, capped by the amount assumed paid in the tenth year. The excess is assumed to be paid in the twelfth year, and it is capped at the same limit. The remaining excess is assumed to be paid in the thirteenth year. We continue in this fashion through the fifteenth year. The remaining excess is assumed to be paid in the sixteenth year, with no limit.

The cumulative percentage paid is $94.949369 \%$ for the ninth year (2001) and $98.0036 \%$ for the tenth year (2000). ${ }^{32}$ The amount assumed paid in the tenth year is $98.0036 \%-94.949368 \%=3.06692 \%$. The amount unpaid after 10 years is $100 \%-98.0036 \%=1.9964 \%$. Since $1.9964 \%$ is less than $3.06692 \%$, the $1.9964 \%$ is assumed to be paid in the eleventh year.
.....Discounting computations use the IRS loss reserve discount rate.

Column 6, the percentage unpaid at the end of the accident year, is the complement of the cumulative percentage paid. For accident year 2009, the cumulative percentage paid is $35 \%$, and the percentage unpaid at the end of the accident year is $100 \%-35 \%=65 \%$.

Column 7 shows the discounted percentage of losses unpaid at the end of the accident year, assuming that all losses are paid at mid-year. We show both an iterative method, working back from the oldest accident year, and a formula method. ${ }^{33}$

From the first row (accident year 2000 in the 2009 Schedule P), $2 \%$ of incurred losses are assumed paid in the middle of the $11^{\text {th }}$ year, so the discounted value at the preceding year end is $2 \% /(1.070)^{0.5}=1.93 \%$. This percentage applies to accident year 2011 at 10 years of maturity (tax year 2020).

Going backward in accident years in Schedule P corresponds to going forward in tax years. The most recent accident year in the exhibit is 2009; the computed payment pattern is used for accident year 2011. Accident year 2000 in Schedule $P$ is 9 years earlier than accident year 2009, so it is used for tax year 2011+9 = 2020 .

Iterative method: The discounted losses unpaid at the end of the $9^{\text {th }}$ year $=$
the losses assumed paid in the middle of the $10^{\text {th }}$ year discounted for half a year to the end of the $9^{\text {th }}$ year

+ the discounted losses unpaid at the end of the $10^{\text {th }}$ year, discounted for one year to the end of the $9^{\text {th }}$ year.
$=3.07 \%$ of accident year 2011 losses assumed paid in the middle of the tenth year ( $7 / 1 / 2020$ ) discounted for half a year to $12 / 31 / 2019=3.07 \% / 1.070^{0.5}=2.97 \%$
+ the discounted percentage of accident year 2011 losses unpaid at the end of the tenth year (12/31/2020), discounted for a full year $=1.93 \% / 1.070=1.80 \%$.

The sum of the two pieces is $4.77 \%$. We continue in this iterative fashion for all accident years.
Formula method: Instead of iterating, we discount each mid-year payment to the appropriate valuation date. The formula for the 2009 accident year in the Schedule P exhibit, which corresponds to accident year 2011 valued at $12 / 31 / 2011$, is

$$
\left(13.00 \% \div 1.07^{0.5}\right)+\left(11.02 \% \div 1.07^{1.5}\right)+\ldots+\left(3.07 \% \div 1.07^{8.5}\right)+\left(2.00 \% \div 1.07^{9.5}\right)=52.26 \%
$$

.....Discount factors are ratios of discounted to undiscounted reserves.
The paid loss percentages above are ratios to incurred losses; discount factors are ratios to unpaid losses. Column 8 , the loss reserve discount factors, are the discounted percentage of unpaid losses at the end of each year divided by the undiscounted percentage at that time.

For accident year 2009 in Schedule P, the discount factor is $52.26 \% / 65.00 \%=80.3944 \%$, which applies to accident year 2011 valued at 12/31/2011 (tax year 2011). If the accident year 2011 undiscounted reserves at $12 / 31 / 2011$ are $\$ 450,000$, the discounted reserves are $\$ 450,000 \times 80.3944 \%=\$ 361,775$.

The loss reserve discount factor in the preceding row, $81.6659 \%$, applies to the accident year 2011 reserves on $12 / 31 / 2012$. If the year end 2012 reserves for accident year 2011 are $\$ 350,000$, the discounted reserves are $\$ 350,000 \times 81.6659 \%=\$ 281,380$.

Loss payments are extended past the tenth year by formula.
Schedule $P$ shows 10 year loss triangles. Actuaries generally estimate payment patterns after the end of the loss triangle by fitting historical values to a mathematical curve (such as exponential decay). Tax accounting uses formulas, leaving nothing to discretion, lest insurers select parameters to avoid paying taxes.

Exhibit 11 (Illustration B) shows the extension of the loss payment pattern beyond the eleventh year. Amounts unpaid at the end of ten years are assumed to be paid in the eleventh year, capped by the amount assumed to be paid in the tenth year. The capping continues through the $15^{\text {th }}$ year; all remaining unpaid losses are assumed to be paid in the $16^{\text {th }}$ year.

Suppose $90.90 \%$ is paid within 10 years and $88.10 \%$ is paid within nine years, implying that $90.90 \%-88.10 \%$ $=2.80 \%$ is paid in the tenth year. The amounts assumed paid in the $11^{\text {th }}, 12^{\text {th }}$, and $13^{\text {th }}$ years are also $2.80 \%$. After thirteen years, $9.10 \%-3 \times 2.8 \%=0.70 \%$ remains unpaid and is assumed to be paid in the $14^{\text {th }}$ year.

The figures in Exhibit 4 are from the 2009 Annual Statement, Schedule P, Part 1H (other liability), of a insurer that elected to use its own loss payment pattern to discount reserves for accident year 2011.

Exhibit 4: Paid and Incurred Losses from 2009 Schedule P, Part 1H (long-tailed line of business)

| Accident | Losses + LAE | Losses + LAE |
| :---: | :---: | :---: |
| $\frac{\text { Year }}{\text { Prior }}$ | $\underline{\text { Paid }}$ | $\underline{\text { Incurred }}$ |
| 2000 | 235,000 | 250,000 |
| 2001 | 50,000 | 55,500 |
| 2002 | 55,000 | 62,000 |
| 2003 | 60,000 | 70,000 |
| 2004 | 65,000 | 80,000 |
| 2005 | 70,000 | 96,000 |
| 2006 | 65,000 | 103,000 |
| 2007 | 60,000 | 115,000 |
| 2008 | 50,000 | 125,000 |
| 2009 | 35,000 | 140,000 |

The 60 month average of the federal mid-term rate, from $1 / 12006$ through $12 / 1 / 2010$, is $7.0 \%$ per annum.
We determine 15 discount factors. The first ten discount factors are used for valuation dates 12/31/2011 through $12 / 31 / 2020$; the $11^{\text {th }}$ through $15^{\text {th }}$ factors are used at valuation dates $12 / 31 / 2021$ through $12 / 31 / 2025$ as part of the composite discount factor for accident years more than 10 years old.

The amount assumed paid in the $11^{\text {th }}$ year is capped by the amount assumed paid in the $10^{\text {th }}$ year:

- $90.09 \%-88.71 \%=1.38 \%$ of incurred losses are assumed to be paid in the tenth year.
- The amount unpaid after 10 years is $100 \%-90.09 \%=9.91 \%$ of the incurred losses.
- $\Rightarrow 1.38 \%$ is assumed to be paid in the eleventh year; the excess is rolled forward and capped each year.

The amount assumed paid in each of the five years following the tenth year is the lesser of (i) the amount unpaid at the end of the previous year and (ii) the $1.38 \%$ cap. Whatever remains after 15 years is assumed paid in the $16^{\text {th }}$ year, even if it exceeds the $1.38 \%$ cap. Here, $9.91 \%-5 \times 1.38 \%=3.01 \%$ remains unpaid after 15 years, so $3.01 \%$ is assumed paid in the sixteenth year. ${ }^{34}$

The iterative method first computes the discounted percentage unpaid at the year end before the final loss payment. The loss payment pattern here lasts 16 years, so we begin with the end of the fifteenth year:
$3.01 \%$ of the accident year 2011 incurred losses are assumed paid in the middle of the $16^{\text {th }}$ year, or July 1,2026 . The discounted loss reserve at the end of the $15^{\text {th }}$ year $(12 / 31 / 2025)$ is $3.01 \% / 1.070^{0.5}=2.91 \%$.

The discounted percentage unpaid at the end of the $14^{\text {th }}$ year $=$
the $2.91 \%$ discounted percentage unpaid at the end of the $15^{\text {th }}$ year discounted for one more year

+ the $1.38 \%$ of the incurred losses assumed to be paid on July 1 of the $15^{\text {th }}$ year discounted for half a year
$=2.91 \% / 1.070+1.38 \% / 1.070^{0.5}=4.05 \%$.
Alternatively, we calculate each discounted percentage unpaid by formula. For the 2011 valuation date for the 2011 accident year, the discounted percentage unpaid is

$$
\left(16.67 \% \div 1.07^{0.5}\right)+\left(15.00 \% \div 1.07^{1.5}\right)+\ldots+\left(1.38 \% \div 1.07^{13.5}\right)+\left(3.01 \% \div 1.07^{14.5}\right)=71.32 \%
$$

Loss payment patterns
Actuaries often assume that loss payment patterns follow exponential decay: the ratio of losses paid in year $t$ to losses paid in year ( $t-1$ ) is $P .{ }^{35}$ With a discount rate of $r$ and losses of $Z$ paid in year $t$ :


The discount factor depends on the decay rate $P$ and the interest rate $r$, not on the maturity of the losses.
In practice, loss portfolio size contain a variety of claim types, some fast settling with discount factors close to $100 \%$ and some slow settling with lower discount factors. The overall discount factor is a weighted average of the discount factors for each claim type. As claims mature, the fast settling claims leave the portfolio and the slow settling claims remain, so the overall discount factor decreases. ${ }^{36}$

If claims are paid evenly over a fixed period of $N$ years, with losses of $Z$ paid in year $t$ :

```
undiscounted losses = Z }\times1/\textrm{N}\times\mp@subsup{\sum}{j=1 to N-t}{}\mp@subsup{P}{}{j
discounted losses =
```



The discount factors increase steadily for the $N$ years.
The IRS loss development factors follow these two patterns. For the first several years, the discount factors decrease, reflecting the closure of fast settling claims and increasing percentage of slow settling claims. But the IRS assumes even payments for the tail, so the discount factors increase at later maturities.

## Negative discount factors

Negative loss payments reflect data errors, reinsurance, or salvage and subrogation. ${ }^{37}$ If the reinsurance (or the salvage and subrogation) is coded to the same accounting period as the associated paid loss, negative loss payments should not occur. If the loss payments are non-negative, so are the discount factors.

The IRS incremental payments are differences of independent accident years. Random loss fluctuations may cause negative payments, especially at late maturities, when expected cumulative loss payments do not differ much by year. They are most problematic (i) when they occur in the tenth year, causing a negative cap for the assumed payments in the following years, or (ii) when they cause a negative loss reserve discount factor.

Negative caps on the extended payments after the $10^{\text {th }}$ year are revised to the average of the assumed loss payments in the three oldest years. ${ }^{38}$

Illustration: Suppose the 2009 Schedule P has the cumulative paid losses and incurred losses in Exhibit 5.
Exhibit 5: Negative Assumed Loss Payment in the Tenth Year
$\left.\begin{array}{|lccccc|}\hline & \begin{array}{c}\text { Paid Loss }+ \\ \text { Accident Year } \\ \text { (1) }\end{array} & \begin{array}{c}\text { Incurred Loss } \\ \text { (2) }\end{array} & \begin{array}{c}\text { Lae } \\ \text { (3) }\end{array} & \begin{array}{c}\text { Cumulative } \\ \text { Paid/incurred } \\ \text { Ratio: (4) }\end{array} & \begin{array}{c}\text { Incremental } \\ \text { Paid/incurred } \\ \text { Ratio: (5) }\end{array}\end{array} \begin{array}{c}\text { Undiscounted } \\ \text { Percentage } \\ \text { Unpaid: (6) }\end{array}\right]$

The negative assumed payment in the oldest accident year (2000) stems from a random fluctuation: the high paid to incurred ratio in the ninth year (2001), reflecting fewer claims remaining open than expected. The revised cap for the $11^{\text {th }}$ and subsequent years is the average assumed loss payment in the three oldest accident years: $1 / 3 \times(-3.64 \%+9.47 \%+5.92 \%)=3.92 \% .^{39}$
.....Negative discount factors are replaced by interpolated figures.
Discount factors show the present values of unpaid losses. If the discount factor is $80 \%$ and unpaid losses are $\$ 100,000$, a risk-neutral investor would be indifferent between receiving $\$ 80,000$ now and the $\$ 100,000$ actual cash flow. Negative discount factors have no comparable meaning. A discount factor of $-40 \%$ cannot mean that investors are indifferent between paying $\$ 40,000$ now and receiving the $\$ 100,000$ actual cash flow.

Negative assumed loss payments at late maturities sometimes cause negative discount factors (though most do not). To avoid unreasonable figures, the negative discount factor is replaced by a linear interpolation between the nearest positive discount factors on both sides.

Illustration: If the computed discount factors for three consecutive maturities are $+80 \%,-35 \%$, and $+85 \%$, the negative discount factor of $-35 \%$ is replaced by the interpolated factor of $+80 \%+1 / 2(85 \%-80 \%)=82.5 \%$. If the computed discount factors for four consecutive maturities are $+70 \%,-35 \%,-45 \%$, and $+85 \%$, the negative discount factors of $-35 \%$ and $-45 \%$ are replaced by the interpolated factors of $+70 \%+2 / 3(85 \%-$ $70 \%)=80 \%$ and $+70 \%+1 / 3(85 \%-70 \%)=75 \%$.

Most negative assumed loss payments produce positive but unreasonably low discount factors, which cause large tax liabilities in one year followed by tax refunds in the next year. This pattern is submerged within the other tax liabilities and tax refunds of the company.

Illustration: The expected loss reserves for accident year 20X1 are $\$ 50$ million, $\$ 45$ million, and $\$ 40$ million at year-end 20X7, 20X8, and 20X9, with expected payments of $\$ 5$ million in each year. The statutory incurred loss from runoff of the reserves is zero in each year. If the loss reserve discount factors are $80 \%, 10 \%$, and $85 \%$, the tax basis incurred losses are shown in Exhibit 6.

Exhibit 6: Unreasonable Loss Reserve Discount Factors (dollars in millions)

| Calendar Year | Paid Loss | Change in Loss Reserve | Incurred Loss |
| :---: | :---: | :---: | :---: |
| $20 \times 8$ | $\$ 5$ | $\$ 45 \times 10 \%-\$ 50 \times 80 \%=-\$ 35.5$ | $-\$ 30.5$ |
| $20 \times 9$ | $\$ 5$ | $\$ 40 \times 85 \%-\$ 45 \times 10 \%=\$ 29.5$ | $\$ 34.5$ |

Exhibits 12 and 13 illustrate this. Exhibit 12 uses the same other liability illustration worked out above, with a change in the paid losses for accident year 2002 from $\$ 60,000$ to $\$ 69,000$. The incremental paid to incurred ratio for accident year 2001 becomes $-9.86 \%$, the discounted loss reserve for accident year 2002 becomes $-1.36 \%$, and the loss reserve discount factor for accident year 2002 is $-95 \%$. The negative loss reserve discount factor is replaced a positive factor of $1 / 2 \times(82.5189 \%+77.4439 \%)=79.9814 \%$.

Exhibit 13 shows the same scenario, but the accident year 2002 paid losses are changed to $\$ 68,000$, not $\$ 69,000$. The incremental paid to incurred ratio for accident year 2001 becomes $-8.43 \%$, the discounted loss reserve for accident year 2002 becomes $0.02 \%$, and the loss reserve discount factor for accident year 2002 is less than $1 \%(0.6645 \%)$. This scenario is also unreasonable, but it is retained by the IRS rules.
.....Composite discount factors are used for prior years row.
The discount factors are applied to the unpaid losses in the Annual Statement by accident year and maturity. Schedule $P$ shows ten accident years. Earlier accident years, with unpaid losses at later maturities, are shown in the Schedule $P$ prior years row. They are not separated by accident year, so the accident year's discount factors for the eleventh through fifteenth valuation dates can not be applied. Instead, a composite discount factor is used for all losses unpaid more than ten years since the accident date.

The IRS discounting procedure assumes that all losses are paid no later than the $16^{\text {th }}$ year, implying that the prior years row in Schedule $P$ has reserves that will be paid in the $12^{\text {th }}$ through the $16^{\text {th }}$ year, which use the discount factors for the $11^{\text {th }}$ through $15^{\text {th }}$ maturities. The composite discount factor is an average of discount factors from the five accident years whose losses are assumed to comprise the reserves in the prior years row. We explain the calculation of the composite discount factor by illustration.

Illustration: We compute up to 15 reserve discount factors for accident year 2009 from the 2007 Schedule P. The first ten discount factors apply to accident year 2009 unpaid losses as shown in the Schedule $P$ exhibits for the 2009 through 2018 Annual Statements. For the 2019 Annual Statement, the accident year 2009 unpaid losses are commingled with the unpaid losses from earlier accident years in the Schedule P prior years row. The discounting procedure assumes that all losses are paid by the $16^{\text {th }}$ year, so we assume that the reserves in the prior years row in the 2019 Schedule P are for losses from accident years 2005 through 2009. The composite discount factor is an average of the accident year 2005 through 2009 discount factors at maturities of 15 years through 11 years. ${ }^{40}$ Suppose these five discount factors are as shown in Exhibit 7.

Exhibit 7: Composite Discount Factor

| Accident <br> Year (1) | Maturity (2) | Undiscounted <br> Reserve (3) | Discounted <br> Reserve (4) | Discount <br> Factor (5) |
| :--- | :---: | :---: | :---: | :---: |
| 2005 | 15 years | $5.0 \%$ | $4.8 \%$ | $96.9 \%$ |
| 2006 | 14 years | $7.2 \%$ | $6.8 \%$ | $93.9 \%$ |
| 2007 | 13 years | $9.1 \%$ | $8.3 \%$ | $91.0 \%$ |
| 2008 | 12 years | $11.7 \%$ | $10.3 \%$ | $88.2 \%$ |
| 2009 | 11 years | $13.3 \%$ | $11.4 \%$ | $85.4 \%$ |
| Total |  | $46.3 \%$ | $41.6 \%$ | $89.8 \%$ |

The discount factors in column 5 are ratios of the discounted reserves in column 4 to the undiscounted reserves in column $3 .{ }^{41}$ The composite discount factor for the prior years row is the ratio of the total discounted reserves to the total undiscounted reserves. ${ }^{42}$

Illustration: We determine between 10 and 15 discount factors for accident year 2011. The first ten discount factors are used for valuation dates $12 / 31 / 2011$ through $12 / 31 / 2020$. The last five discount factors are used in the composite discount factor for the prior years row for valuation dates 12/31/2021 through 12/31/2025. Exhibit 8 shows the discount factors and the associated valuation dates and tax years.

Exhibit 8: Valuation Dates for Loss Reserve Discount Factors

| Discount <br> Factor for | Accident Year | Individual// <br> Composite | Tax <br> Year |
| :---: | :---: | :---: | :---: |
| 1 year | 2011 | individual | 2011 |
| 2 years | 2011 | individual | 2012 |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| 10 years | 2011 | individual | 2020 |
| 11 years | 2011 and prior | composite | 2021 |
| 12 years | 2012 and prior | composite | 2022 |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| 15 years | 2015 and prior | composite | 2025 |

The tax year is the calendar year for which taxes are computed. Each tax year uses two discount factors: one for the ending reserves and one for the beginning reserves. The maturity shown in the first column ("discount factor for") is the maturity of the ending reserves.

The first ten discount factors, which apply to accident year 2011 only, are used at valuation dates from 1 year to 10 years from the beginning of the accident year, corresponding to tax years 2011 through 2020. For later valuation dates, the discount factor for accident year 2011 is combined with discount factors for other years to form a composite factor.

Illustration: For tax year 2022, the accident year 2011 discount factor for a maturity of 12 years (computed from the 2009 Schedule $P$ ) is averaged with the accident year 2012 discount factor for a maturity of 11 years (computed from the 2010 Schedule P), the accident year 2010 discount factor for a maturity of 13 years (computed from the 2008 Schedule P), the accident year 2009 discount factor for a maturity of 14 years
(computed from the 2007 Schedule P), and the accident year 2008 discount factor for a maturity of 15 years (computed from the 2006 Schedule P). The composite discount factor is applied to the accident year reserves in the 2012 and prior year line in the 2022 Schedule P.

Insurers elect every five years whether to use their own Schedule P data.
The Treasury determines line of business loss payment patterns every five years from aggregate Schedule P data. The first loss payment patterns were determined in late 1986 and early 1987 for the 1987-1991 tax years (for review by Congress), using the most recently published (1986) edition of Best's Aggregates and Averages, containing data from the 1985 Annual Statements. The Treasury redetermines these loss payment patterns for the five year periods beginning in 1992, 1997, 2002, 2007, ... (1992-1996; 1997-2001; ...).

In these determination years $(1992,1997, \ldots)$, insurers elect to use either discount factors developed by the Treasury, based on industry aggregate data from Best's Aggregates and Averages, or discount factors derived from their own Schedule $P$ data. The election is made with the insurer's tax filing for the determination year; ${ }^{43}$ it applies to that year and to the next four years. ${ }^{44}$ If the insurer uses its own data to determine the loss payment pattern, the patterns are updated each year. ${ }^{45}$ In contrast, industry loss payment patterns are vintaged and updated in determination years. Determination years end in a 2 or a 7 , and they use aggregate industry data for statement dates ending in a 0 or a 5 . For determination year 20X2, data as of 12/31/20X0 are used. For determination year 20X7, data as of 12/31/20X5 are used.

The election applies to all lines of business (except international and reinsurance); ${ }^{46}$ insurers cannot elect to use their own data for one line and industry data for other lines. ${ }^{47}$

The industry loss payment patterns are determined every five years, but the discount rate is recomputed each year. The discount factors change each year, since the discount rate changes.

## Loss payment lags

Loss payment lags differ among insurers writing the same line of business. Some classes or policies have short or long lags, regardless of the insurer. Classes with less severe claims, such as nursing classes in workers' compensation or retail stores in general liability, have shorter loss payment lags; classes with more severe claims, such as construction crews in workers' compensation, have longer loss payment lags. Larger claims generally have longer payment lags, so high policy limits or high deductibles result in longer lags.

A longer loss payment lag causes a lower reserve discount factor, a lower present value for incurred losses, greater underwriting income, and a higher tax liability. Insurers with shorter average loss payment lags have incentives to use their own data for loss payment patterns.

## Reserve margins

Insurers who hold fair value loss reserves often add reserve margins. IFRS 4 (to be re-issued as IFRS 17) mandates that risk margins be added to the present values of future loss cash flows. The AAA Standards of Practice look favorably on loss reserve risk margins added to discounted reserves.

Reserve margins increase reported losses, reducing underwriting income and the associated tax liability. The IRS examines insurers' reported losses to eliminate reserve margins, thereby raising the tax liability. Reserve margins may be suggested by

- Reserve indications higher than the rote application of the reserving method.
- Annual Statement booked reserves higher than the actuarial reserve indications.
- Favorable reserve development in Schedule P, Part 2.
- Average paid losses lower than the held reserves.
- IRS reserve indications lower than those of the insurer.

In the mid-1980s, interest rates on Treasury securities were about $7 \%$ or $8 \%$, and few insurers added reserve margins to the undiscounted Annual Statement reserves. In the 2000s and 2010s, these interest rates are low: the 60 moving average mid-term rate used for the 2016 loss reserve discount factors is $1.47 \%$. More insurers are added risk margins to their statement reserves, and the IRS often audits the tax returns and contests the reserve margin. The IRS may impose a tax penalty if an audit discloses a revenue margin. ${ }^{48}$

The NAIC Statement of Actuarial Opinion applies to total reserves for all accident years and lines of business; a deficiency in one line or accident year may be offset by redundancies in other lines or accident years. The IRS examines the lines of business where it suspects a reserve margin; redundancies in this line are not offset by deficiencies in other lines.

## Two-year lines of business

Several lines of business show two accident years and a prior years row in Schedule $P$, not ten accident years: Auto Physical Damage, Fidelity/Surety, Financial Guaranty/Mortgage Guaranty, Miscellaneous Casualty, Other (Including Credit), Special Property (Fire, Allied Lines, Inland Marine, Earthquake, Burglary, and Theft), and Warranty. The assumed loss percentage paid for the latest Schedule $P$ accident year is computed the same way as for the ten year lines of business. The losses remaining unpaid after two years are assumed to be paid evenly over the next two years (half in each year).

Illustration: The 2013 Schedule P shows cumulative losses paid for auto physical damage as in column (2) of Exhibit 9. The 60 month moving average mid-term rate for accident year 2015 is $1.68 \%$.

Exhibit 9: Loss Payment Pattern for Auto Physical Damage
(Data from Schedule P, Part 1, from the 2013 Annual Statement)

| Accident Year <br> (1) | Cumulative <br> Percentage <br> Paid (2) | Incremental <br> Percentage <br> Paid (3) | Unpaid Losses <br> at Year End <br> (4) | Discounted <br> Unpaid Losses <br> at Year End (5) | Discount <br> Factor (6) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2012 | $99.7478 \%$ | $9.4821 \%$ | $0.2522 \%$ | $0.2480 \%$ | $98.3512 \%$ |
| 2013 | $90.2657 \%$ | $90.2657 \%$ | $9.7343 \%$ | $9.6474 \%$ | $99.1071 \%$ |

Column 2, cumulative percentage paid, is cumulative paid losses divided by incurred losses.
Column 3, incremental percentage paid, is the first difference of column 2.
Column 4, unpaid losses at year end, is the complement of column 2.
Column 5, discounted unpaid losses at year end, is computed by the iterative method or the formula method:

- $0.2522 \% \times 1 / 2 \times\left(1 / 1.0168^{0.5}+1 / 1.0168^{1.5}\right)=0.2480 \%$
- $0.2480 \% / 1.0168^{1.0}+(9.7343 \%-0.2522 \%) / 1.0168^{0.5}=9.6474 \%$

Column 6, discount factor, is column 5 divided by column 4.
The discount factor for the two year maturity depends on the discount rate only, not on the Schedule $P$ loss payment pattern, since the IRS assumes that the unpaid losses after two years are paid half in the third year and half in the fourth year: $1 / 2 \times\left(1 / 1.0168^{0.5}+1 / 1.0168^{1.5}\right)=98.3512 \%$

## Required experience for an election

IRC $\S 846(e)(4)(A)$ says that the Secretary of the Treasury may prescribe "regulations providing that a taxpayer may not make an election ... if [it] does not have sufficient historical experience for the line of business to determine a loss payment patter." The 1986 conference reports and the 1988 Treasury regulation 88-100 interpreted this provision to mean that an insurer whose reserves in a line of business were smaller than those
of $90 \%$ of other nsurers may not use its own data to determine the loss payment patterns. ${ }^{49}$ Small insurers complained that this provision discriminated against them. In 1991, the Secretary of the Treasury revoked this interpretation. Instead, the insurer must have data for all ten accident years in Schedule $P$ to use its own data for that line of business. ${ }^{50}$

The adequacy of an insurer's loss reserves affects its election to use its own data. An insurer with less adequate reserves than the industry average may benefit from using its own data.

Illustration: In 20X9, the industry Schedule P for a given line of business shows accident year 20X9 cumulative paid losses of $\$ 100$ million and incurred losses of $\$ 400$ million, indicating that $25 \%$ of losses are paid in the first 12 months. Insurer ABC, which holds less adequate loss reserves, shows $\$ 3$ million of accident year 20X9 cumulative paid losses and $\$ 10$ million of incurred losses, indicating that $30 \%$ of losses are paid in the first 12 months. Insurer ABC seems to pay its losses more rapidly, so its discount factor should be closer to unity, its offset to taxable income should be larger, and its tax liability should be smaller. In truth, insurer ABC may have the same loss payment pattern as the industry, but it may be holding less adequate loss reserves.

## Anticipated salvage and subrogation must be disclosed if it is netted.

The loss reserves that are an offset to taxable income must be net of anticipated salvage and subrogation. ${ }^{51}$ Unless the insurer discloses that unpaid losses in Schedule $P$ are net of anticipated salvage and subrogation, the IRS assumes they are gross and requires a reduction for the anticipated amounts. Schedule P, Part 1, column 23, provides this disclosure by accident year and by line of business. ${ }^{52}$

Discounted future salvage and subrogation using discount factors determined by the Treasury is subtracted from unpaid losses. Insurers may elect to use their own discount factors for loss reserves, but they must use the Treasury discount factors for anticipated salvage and subrogation. ${ }^{53}$

The sequence for determining the loss reserves offset to taxable income is as follows:

- Total net losses and expenses unpaid are taken from Schedule P, Part 1, column 24.
- The salvage and subrogation anticipated from Schedule P, Part 1, column 23, is added.
- The tabular discounts for loss reserves from Note 27 are added to get the unpaid losses gross of all discounts and of anticipated salvage and subrogation. The Schedule P, Part 1, loss reserves are already gross of the non-tabular discounts in columns 32 and 33.
- Loss reserves are discounted using either (i) the industry loss reserve discount factors published by the Treasury or (ii) the company's own loss reserve discount factors, depending on the election made by the company in the most recent determination year.
- The anticipated salvage and subrogation is discounted with the Treasury discount factors.
- The discounted anticipated salvage and subrogation is subtracted from the discounted loss reserves to give the discounted reserves net of anticipated salvage and subrogation. The change in these discounted reserves is the loss reserve offset to taxable income.

Tax years, accident years, and maturities
The IRS procedure uses Schedule $P$ accident years to derive discount factors for accident years (at beginning and ending maturities) in future tax years. The description of loss reserve discounting in the previous sections begin with the Schedule $P$ accident years and the mid-term discount rates to derive the tax basis incurred losses in future tax years. This section starts with the tax year incurred losses and shows the Schedule P accident years and mid-term discount rates from which it is derived.

- The tax year is the calendar year for which taxes are being paid. The statutory calendar year incurred loss is the paid loss plus the change in reserves. The tax year incurred loss is the paid loss plus the change in discounted reserves, including the accrual of discount from previous accident years.
- The incurred loss is computed separately for each accident year, to which two discount factors apply: one at the beginning of the tax year and one at the end of the tax year. The discount factors for different accident years at the same date use different discount rates and may use different loss payment patterns.
- The Schedule P accident years discussed in this reading are used to derive loss payments; they are not the accident years from which incurred losses are derived. Each set of ten Schedule P accident years provides discount factors for one future accident year.
- Mid-term discount rates are calendar month items: the annual rates for mid-term Treasury securities in a given month. An average of 60 calendar month discount rates is frozen and applied to accident year incurred losses at future maturities.

Illustration: An insurer files its corporate income tax return in March 2020 for tax year 2019.
The incurred loss offset to taxable income $=$

- Losses paid in 2019
-     + discounted loss reserves at year-end 2019
-     - discounted loss reserves at year-end 2018
-     - anticipated discounted salvage and subrogation at year-end 2019
-     + anticipated discounted salvage and subrogation at year-end 2018. ${ }^{54}$

This illustration focuses on loss reserve discounting (ignoring salvage and subrogation).
The loss reserve discount factors are by accident year, so we separate the discounted loss reserves at yearend 2018 and 2019 into their accident year components. The discounted loss reserves at year-end $2019=$

| Undiscounted <br> Reserves at Year- <br> end 2019 for | $\times$ Discount Factor for <br> this Accident Year <br> derived from | at Accident Year <br> Maturity of |
| :---: | :---: | :---: |
| Accident Year | Schedule P of |  |
| 2019 | 2017 | 1 year |
| 2018 | 2016 | 2 years |
| 2017 | 2015 | 3 years |
| $\ldots$ | $\ldots$ | $\ldots$ |

The table above assumes the insurer derives loss payment patterns from its own Schedule $P$ data. If it uses the industry pattern, accident years 2017-2019 use the 2015 Schedule $P$, accident years 2012-2016 use the 2010 Schedule P, and accident years 2010-2011 use the 2005 Schedule P. We examine the 2017 accident year, which uses the 2015 Schedule $P$ whether the insurer uses its own data or aggregate data.

The discount factors for accident year 2017 in tax year $2019=$
the discounted loss reserves for the 2015 Schedule P at a maturity of 3 years
$\div$ the undiscounted loss reserves for the 2015 Schedule $P$ at a maturity of 3 years
An accident year has losses paid in several calendar years. The IRS loss payment pattern computes assumed percentages paid by maturity from differences in cumulative percentages paid in adjoining accident years. The percentages paid are not the amounts actually paid in that maturity for either the Schedule $P$ accident year or for the accident year for which tax basis incurred losses are computed. Consider two payment periods of accident year 2017: 12 to 24 months (calendar year 2018) and 24 to 36 months (calendar year 2019).

The assumed percentage paid for accident year 2017 from 12 to 24 months =
The cumulative percentage paid for accident year 2014 from 0 to 24 months

- The cumulative percentage paid for accident year 2015 from 0 to 12 months

The assumed percentage paid for accident year 2017 from 12 to 24 months =
The cumulative percentage paid for accident year 2013 from 0 to 36 months

- The cumulative percentage paid for accident year 2014 from 0 to 24 months
where all figures above are from the 2015 Schedule P.
The cumulative percentages paid as ratios of cumulative dollars paid to dollars incurred:
The cumulative percentage paid for accident year 2013 from 0 to 36 months $=$

The cumulative paid losses for accident year 2013 in the 2015 Schedule P Part 1
$\div \quad$ The incurred losses for accident year 2013 in the 2015 Schedule P Part 1.
The cumulative percentage paid for accident year 2014 from 0 to 24 months $=$
The cumulative paid losses for accident year 2014 in the 2015 Schedule P Part 1
$\div \quad$ The incurred losses for accident year 2014 in the 2015 Schedule P Part 1.
The cumulative percentage paid for accident year 2015 from 0 to 12 months =
The cumulative paid losses for accident year 2015 in the 2015 Schedule P Part 1
$\div \quad$ The incurred losses for accident year 2015 in the 2015 Schedule P Part 1.
Each accident year in tax year 2019 uses a different discount rate, formed from the 60 month moving average of federal mid-term rates in the five years preceding the accident year. The discount rate and the incremental percentages paid give the ratio of discounted reserves to ultimate losses. Dividing this ratio by the ratio of full value reserves to ultimate losses gives the loss reserve discount factor.

Actuaries often assume that loss payment patterns follow exponential decay: the ratio of losses paid in year $t$ to losses paid in year (t-1) is $P$. With a discount rate of $r$ and losses of $Z$ paid in year $t$ :

The undiscounted losses are $Z \times \sum_{j=1}^{\infty} P^{j}$.
The discounted losses are $Z \times \sum_{j=1}^{\infty}\left(\frac{P}{1+r}\right)^{j}$.
The discount factor (discounted losses divided by undiscounted losses) depends on the decay rate $P$ and the interest rate $r$, not on the maturity of the losses.

In practice, loss portfolio size contain a variety of claim types, some fast settling with discount factors close to $100 \%$ and some slow settling with lower discount factors. The overall discount factor is a weighted average of the discount factors for each claim type. As claims mature, the faster settling claims leave the portfolio and the slower settling claims remain, so the overall discount factor decreases.

If claims are paid evenly over a fixed period of $N$ years, with losses of $Z$ paid in year $t$ :

The undiscounted losses are $Z \times \frac{1}{N} \sum_{j=1}^{N-t} P^{j}$.
The discounted losses are $Z \times \frac{1}{N} \sum_{j=1}^{N-t}\left(\frac{P}{1+r}\right)^{j}$.
The decay rate $P$ is less than one, and the interest rate $r$ is more than zero, so $1+r$ is more than one, and $P /(1+r)$ is less than one. The ratio of discounted loss to undiscounted loss decreases as the maturity lengthens, so as claims approach the payment date, the ratio of discounted loss to undiscounted loss increases. The discount factors increase steadily for the $N$ years.

The IRS loss reserve discount factors have these two patterns. At early maturities, the discount factors decrease, reflecting the closure of fast settling claims and increasing percentage of slow settling claims. But the IRS assumes even payments for the tail, so the discount factors increase at late maturities.

- Accident year for tax computation
- Taxable income for firms other than insurers is computed from GAAP statements, which show calendar year income, not accident year income. For insurers, taxable income is computed from statutory statements (which also show calendar year income), but the incurred loss offset to taxable income is not the calendar year incurred losses in the income statement. Rather, the undiscounted incurred loss is the sum of the changes (from the beginning to the end of the year) in the ten accident years and the prior years row in Schedule P. The discounted incurred loss used for taxable income modifies this sum by applying the appropriate IRS loss reserve discount factor to the 22 figures ( 10 individual years plus one prior years row, with beginning and end of the year values for each). The accident years to which the IRS loss reserve discount factors are applied are not the accident years from which the IRS loss reserve discount factors are computed. Tax basis incurred losses are computed by accident year using two discount factors: one at the beginning of the year and one at the end of the year. Discount factors for different accident years at the same date use different discount rates and may use different loss payment patterns
- Discount rate, discount factor, and composite discount factor:
- A discount rate is an interest rate that measures the present value of a cash flow to be received at a future date. Loss reserve discounting uses annually compounded rates, since financial statements show cash flows at annual intervals. An annually compounded rate $r_{\mathrm{a}}$ for an N year duration gives the present value of a cash flow $C$ to be received or paid $N$ years from now as $C /\left(1+r_{a}\right)^{N}$. The federal mid-term rate is determined from Treasury securities with semi-annual cash flows. A semi-annually compounded rate $r_{\mathrm{s}}$ satisfies the relation $\left(1+1 / 2 r_{\mathrm{s}}\right)^{2}=\left(1+r_{\mathrm{a}}\right)$.
- A loss reserve discount factor is the ratio of discounted loss reserves to undiscounted loss reserves. Loss reserve discount factors differ by line of business, by accident year, and by maturity.
- The composite discount factor, which applies to loss reserves in the Schedule $P$ prior years row, is an average of the discount factors in the "extended" portion of the Schedule $P$ computation. The composite discount factor differs by line of business, has a beginning of the year and an end of the year value, and changes each year.
- In determination years (1992, 1997, ...), insurers elect (for that year and to the next four years) to use either discount factors developed by the Treasury, based on aggregate industry data, or discount factors derived from their own data. Patterns derived from the insurer's own data are updated each year; industry patterns are updated only in determination years.
- The federal mid-term rate (AFR) has three types:
- Short-term AFRs: yields of U.S. government T-bills with maturities of three years or less.
- Mid-term AFR: yields of U.S. government Treasury notes and bonds with maturities of more than three years and up to nine years.
- Long-term AFR: yields of U.S. government Treasury bonds with maturities of more than nine years.
- Mid-term rates for 2016 are (the quarterly and monthly rates equal the semi-annual rates in 2016):

Month Annual Semiannual

|  | Jan. | $1.81 \%$ |
| :--- | :--- | :--- |
| Feb. | $1.82 \%$ | $1.80 \%$ |
| March | $1.48 \%$ | $1.47 \%$ |
| April | $1.45 \%$ | $1.44 \%$ |
| May | $1.43 \%$ | $1.42 \%$ |
| June | $1.41 \%$ | $1.41 \%$ |
| July | $1.43 \%$ | $1.42 \%$ |
| Aug. | $1.18 \%$ | $1.18 \%$ |
| Sept. | $1.22 \%$ | $1.22 \%$ |
| Oct. | $1.29 \%$ | $1.29 \%$ |
| Nov. | $1.33 \%$ | $1.33 \%$ |
| Dec. | $1.47 \%$ | $1.46 \%$ |

- IFRS 4 is the IASB accounting standard for insurance contracts. The most recent exposure draft (ED/2013/7) was issued in June 2013. An updated version, to be renamed IFRS 17 (to be effective in 2021), is scheduled to be issued in 2017. The provisions related to loss reserve are:
- The loss reserve discount rates should "be consistent with observable current market prices for instruments with cash flows whose characteristics are consistent with those of the insurance contract, in terms of, for example, timing, currency and liquidity." (The IRS loss reserve discount rates are five year averages, not current market rates, and they use a single "mid-term" duration.)
- The loss reserve risk adjustment "adjusts those future cash flows for the effects of uncertainty about the amount and timing of those cash flows." (The IRS disallows any risk margins or revenue margins.)
- To separate underwriting performance from changes in discount rates that unwind over time, insurers should recognize:
- in profit or loss interest expense determined on an amortised cost basis.
- in other comprehensive income the difference between the carrying amount of the insurance contract measured using the discount rates that were used to determine that interest expense, and the carrying amount of the insurance contract measured using the current discount rates.
- (IRS loss reserve discounting has only the amortization of the interest discount, not the reconciliation to current market rates.)
- A reserve margin is an accounting entry to ensure that the estimated liability supports the actual cash flows.
- The present value of loss cash flows (at a risk-free rate) will support the actual cash flows about half the time and will be insufficient about half the time. (The actual percentages depend on the shape of the loss distribution.) An insurer may add a reserve margin so that the liability supports the actual cash flows in more scenarios.
- Solvency II requires a reserve margin that is an allocation of equity. RBC requires a risk margin that is partly an allocation of equity and partly a replacement of the risk margin in undiscounted reserves that is held as a liability.
- IRS loss reserve discounting includes no risk margin, and reserve margins are not permitted in tax reserves.
- A risk margin adjusts the present value (at a risk-free rate) of a cash flow to its market value.
- IFRS 4 determines the present value of loss cash flows at a risk-free rate and adds a risk margin to bring this value to the assumed market value.
- When the loss is paid, the risk margin is returned to the insurer. But the IFRS 4 risk margin is a liability, like the loss reserve, not an allocation of equity.
- Schedule $P$ accident year: Schedule $P$ shows ten accident years and a prior years row. The loss reserve discount factors for accident year 20XX are computed from the loss payment pattern in the Schedule $P$ for valuation date 20XX-2, which shows accident years 20XX-11 through 20XX-2.
- Tax law consists of the Internal Revenue Code (IRC), IRS regulations and revenue procedures, and Tax Court rulings, all of which affect loss reserve discounting:
- The Internal Revenue Code (legislated by the U.S. Congress) sets the tax law; Section 832 defines insurance company taxable income and Section 846 defines discounted unpaid losses for propertycasualty insurers. Section 846 was new with the 1986 Tax Reform Act.
- IRS revenue procedures prescribe the salvage discount factors and the industry loss payment patterns and loss reserve discount factors for an accident year. For example, revenue procedure 2015-52 prescribes the loss payment patterns and discount factors for the 2015 accident year, and revenue procedure 2015-54 prescribes the salvage discount factors for the 2015 accident year.
- Tax court rulings affect whether insurer reserves have excessive margins.
- The tax year is the period of time covered by a tax return. The 20XX tax year is the calendar year income in the 20XX Annual Statement, modified by the adjustments in the Internal Revenue Code (IRC) - except for the incurred loss deduction, which is an accident year computation (see "accident year for tax computation"). The tax return for the 20XX tax year is filed in March 20XX +1 , and the estimated taxes are paid quarterly in 20XX.

Exhibit 10:illustration A Loss Reserve Discount Factors

| Accident Year (1) | Paid Loss + LAE <br> (2) | Incurred Loss + LAE (3) | Cumulative Paid/Incurred Ratio (4) | Incremental Paid/Incurred Ratio (5) | Undiscounted Percentage Unpaid (6) | Discounted Percentage Unpaid (7) | Loss Reserve Discount Factor (8) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AY + 10 |  |  |  | 2.00\% | 0.00\% | 0.00\% |  |
| 2000 | \$270,000 | \$275,500 | 98.00\% | 3.07\% | 2.00\% | 1.93\% | 96.6735\% |
| 2001 | \$300,000 | \$316,000 | 94.94\% | 2.98\% | 5.06\% | 4.77\% | 94.1800\% |
| 2002 | \$320,000 | \$348,000 | 91.95\% | 3.99\% | 8.05\% | 7.34\% | 91.2271\% |
| 2003 | \$340,000 | \$386,500 | 87.97\% | 4.93\% | 12.03\% | 10.71\% | 89.0399\% |
| 2004 | \$350,000 | \$421,500 | 83.04\% | 6.03\% | 16.96\% | 14.78\% | 87.1281\% |
| 2005 | \$370,000 | \$480,500 | 77.00\% | 7.98\% | 23.00\% | 19.65\% | 85.4281\% |
| 2006 | \$380,000 | \$550,500 | 69.03\% | 10.01\% | 30.97\% | 26.07\% | 84.1740\% |
| 2007 | \$360,000 | \$610,000 | 59.02\% | 11.02\% | 40.98\% | 34.04\% | 83.0660\% |
| 2008 | \$330,000 | \$687,500 | 48.00\% | 13.00\% | 52.00\% | 42.47\% | 81.6659\% |
| 2009 | \$200,000 | \$571,500 | 35.00\% | 35.00\% | 65.00\% | 52.26\% | 80.3944\% |

Exhibit 11:Illustration B Loss Reserve Discount Factors

| Accident Year (1) | Paid Loss + LAE <br> (2) | Incurred Loss + LAE (3) | Cumulative Paid/Incurred Ratio (4) | Incremental Paid/Incurred Ratio (5) | Undiscounted Percentage Unpaid (6) | Discounted Percentage Unpaid (7) | Loss Reserve Discount Factor (8) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AY + 15 |  |  | 100.00\% | 3.01\% | 0.00\% | 0.00\% |  |
| AY + 14 |  |  | 96.99\% | 1.38\% | 3.01\% | 2.91\% | 96.6736\% |
| $A Y+13$ |  |  | 95.61\% | 1.38\% | 4.39\% | 4.05\% | 92.3385\% |
| $A Y+12$ |  |  | 94.23\% | 1.38\% | 5.77\% | 5.12\% | 88.7803\% |
| $A Y+11$ |  |  | 92.85\% | 1.38\% | 7.15\% | 6.12\% | 85.6177\% |
| AY + 10 | \$235,000 | \$250,000 | 91.47\% | 1.38\% | 8.53\% | 7.06\% | 82.7122\% |
| 2000 | \$50,000 | \$55,500 | 90.09\% | 1.38\% | 9.91\% | 7.93\% | 79.9988\% |
| 2001 | \$55,000 | \$62,000 | 88.71\% | 3.00\% | 11.29\% | 8.74\% | 77.4439\% |
| 2002 | \$60,000 | \$70,000 | 85.71\% | 4.46\% | 14.29\% | 11.07\% | 77.4718\% |
| 2003 | \$65,000 | \$80,000 | 81.25\% | 8.33\% | 18.75\% | 14.66\% | 78.1822\% |
| 2004 | \$70,000 | \$96,000 | 72.92\% | 9.81\% | 27.08\% | 21.76\% | 80.3309\% |
| 2005 | \$65,000 | \$103,000 | 63.11\% | 10.93\% | 36.89\% | 29.82\% | 80.8185\% |
| 2006 | \$60,000 | \$115,000 | 52.17\% | 12.17\% | 47.83\% | 38.44\% | 80.3644\% |
| 2007 | \$50,000 | \$125,000 | 40.00\% | 15.00\% | 60.00\% | 47.69\% | 79.4828\% |
| 2008 | \$35,000 | \$140,000 | 25.00\% | 16.67\% | 75.00\% | 59.07\% | 78.7611\% |
| 2009 | \$15,000 | \$180,000 | 8.33\% | 8.33\% | 91.67\% | 71.32\% | 77.8022\% |

Exhibit 12:Other Liability Loss Reserve Discount Factors

| Accident Year (1) | Paid Loss + LAE <br> (2) | Incurred Loss + LAE (3) | Cumulative Paid/Incurred Ratio (4) | Incremental Paid/Incurred Ratio (5) | Undiscounted Percentage Unpaid (6) | Discounted Percentage Unpaid (7) | Loss Reserve Discount Factor <br> (8) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $A Y+15$ |  |  | 100.00\% | 3.01\% | 0.00\% | 0.00\% |  |
| AY + 14 |  |  | 96.99\% | 1.38\% | 3.01\% | 2.91\% | 96.6736\% |
| AY + 13 |  |  | 95.61\% | 1.38\% | 4.39\% | 4.05\% | 92.3385\% |
| $A Y+12$ |  |  | 94.23\% | 1.38\% | 5.77\% | 5.12\% | 88.7803\% |
| AY + 11 |  |  | 92.85\% | 1.38\% | 7.15\% | 6.12\% | 85.6177\% |
| AY + 10 | \$235,000 | \$250,000 | 91.47\% | 1.38\% | 8.53\% | 7.06\% | 82.7122\% |
| 2000 | \$50,000 | \$55,500 | 90.09\% | 1.38\% | 9.91\% | 7.93\% | 79.9988\% |
| 2001 | \$55,000 | \$62,000 | 88.71\% | -8.43\% | 11.29\% | 8.74\% | 77.4439\% |
| 2002 | \$68,000 | \$70,000 | 97.14\% | 15.89\% | 2.86\% | 0.02\% | 0.6645\% |
| 2003 | \$65,000 | \$80,000 | 81.25\% | 8.33\% | 18.75\% | 15.38\% | 82.0371\% |
| 2004 | \$70,000 | \$96,000 | 72.92\% | 9.81\% | 27.08\% | 22.43\% | 82.8251\% |
| 2005 | \$65,000 | \$103,000 | 63.11\% | 10.93\% | 36.89\% | 30.45\% | 82.5297\% |
| 2006 | \$60,000 | \$115,000 | 52.17\% | 12.17\% | 47.83\% | 39.03\% | 81.5980\% |
| 2007 | \$50,000 | \$125,000 | 40.00\% | 15.00\% | 60.00\% | 48.24\% | 80.4018\% |
| 2008 | \$35,000 | \$140,000 | 25.00\% | 16.67\% | 75.00\% | 59.59\% | 79.4482\% |
| 2009 | \$15,000 | \$180,000 | 8.33\% | 8.33\% | 91.67\% | 71.80\% | 78.3276\% |

Exhibit 13: Other Liability Loss Reserve Discount Factors

| Accident Year (1) | Paid Loss + LAE <br> (2) | Incurred Loss + LAE (3) | Cumulative Paid/Incurred Ratio (4) | Incremental Paid/Incurred Ratio (5) | Undiscounted Percentage Unpaid (6) | Discounted Percentage Unpaid (7) | Loss Reserve Discount Factor <br> (8) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AY + 15 |  |  | 100.00\% | 3.01\% | 0.00\% | 0.00\% |  |
| AY + 14 |  |  | 96.99\% | 1.38\% | 3.01\% | 2.91\% | 96.6736\% |
| $A Y+13$ |  |  | 95.61\% | 1.38\% | 4.39\% | 4.05\% | 92.3385\% |
| AY + 12 |  |  | 94.23\% | 1.38\% | 5.77\% | 5.12\% | 88.7803\% |
| $A Y+11$ |  |  | 92.85\% | 1.38\% | 7.15\% | 6.12\% | 85.6177\% |
| $A Y+10$ | \$235,000 | \$250,000 | 91.47\% | 1.38\% | 8.53\% | 7.06\% | 82.7122\% |
| 2000 | \$50,000 | \$55,500 | 90.09\% | 1.38\% | 9.91\% | 7.93\% | 79.9988\% |
| 2001 | \$55,000 | \$62,000 | 88.71\% | -9.86\% | 11.29\% | 8.74\% | 77.4439\% |
| 2002 | \$69,000 | \$70,000 | 98.57\% | 17.32\% | 1.43\% | -1.36\% | -95.3447\% |
| 2003 | \$65,000 | \$80,000 | 81.25\% | 8.33\% | 18.75\% | 15.47\% | 82.5189\% |
| 2004 | \$70,000 | \$96,000 | 72.92\% | 9.81\% | 27.08\% | 22.52\% | 83.1368\% |
| 2005 | \$65,000 | \$103,000 | 63.11\% | 10.93\% | 36.89\% | 30.53\% | 82.7436\% |
| 2006 | \$60,000 | \$115,000 | 52.17\% | 12.17\% | 47.83\% | 39.10\% | 81.7523\% |
| 2007 | \$50,000 | \$125,000 | 40.00\% | 15.00\% | 60.00\% | 48.31\% | 80.5167\% |
| 2008 | \$35,000 | \$140,000 | 25.00\% | 16.67\% | 75.00\% | 59.65\% | 79.5341\% |
| 2009 | \$15,000 | \$180,000 | 8.33\% | 8.33\% | 91.67\% | 71.86\% | 78.3932\% |

1 Tax basis loss adjustment expenses (LAE) are reported on a paid basis since unpaid losses include any unpaid LAE; see IRC §846(f)(2): "The term 'unpaid losses' includes any unpaid loss adjustment expenses shown on the annual statement."

2 The offsetting is exact if the company holds fully discounted reserves (with disclosure), the IRS discount rate equals the investment yield, and the IRS loss payment pattern equals the liquidation pattern of the losses.

3 When the Tax Reform Act was passed, some insurers said they were pre-paying federal income taxes on underwriting income. The modern perspective is that before 1986, the IRS paid part of the loss created by statutory accounting; after 1986, the IRS taxes the economic income.

4 This illustration is not exact, since underwriting income during the policy year is based on written premium and expenses, not on discounted incurred losses, and insurers pay tax on investment income. In addition, insurance markets are competitive, so the tax refund from the IRS lowered premium rates and was received by policyholders, not by insurers.

5 U.S. GAAP has issued a similar exposure draft on insurance contracts, though its implementation is uncertain.

6 The IFRS 4 risk margin are illiquidity premium do not reflect mainstream financial theory. But financial economists do not agree on how risk and liquidity affect market values.

7 IFRS 17, the revised version of IFRS 4, is scheduled to be issued in 2018 and should become effective about two years after the final version is complete. The FASB initially worked with the IASB on IFRS 4 but then issued its won exposure draft on insurance contracts, which is now in abeyance. The NAIC once considered revising statutory accounting to conform more closely with IFRS 4, but no longer intends to change.

8 The actual tax procedure assumes mid-year payments and a longer loss payment pattern.
9 See section 846(b)(2) of the Internal Revenue Code: "Adjustment If Losses Discounted on Annual Statement: If the amount of unpaid losses shown in the annual statement is determined on a discounted basis, and the extent to which the losses were discounted can be determined on the basis of information disclosed on or with the annual statement, the amount of the unpaid losses shall be determined without regard to any reduction attributable to such discounting." The disclosure of non-tabular discounts by accident year and line of business is provided in columns 34 (losses) and 35 (loss adjustment expenses) of Schedule P, Part 1. The disclosure of tabular discounts is shown in note 28 (in the 2001 Annual Statement) to the financial statements, "Discounting of Liabilities for Unpaid Losses or Unpaid Loss Adjustment Expenses."

10 See the Internal Revenue Code $\S 846(\mathrm{a})(3)$ : "In no event shall the amount of the discounted unpaid losses with respect to any line of business attributable to any accident year exceed the aggregate amount of unpaid losses with respect to such line of business for such accident year included on the annual statement."

11 Statutory accounting allows only limited discounting: tabular discounts and exceptional cases of nontabular discounts. For tabular discounts, most insurers use conservative interest rates, such as 3.5\% or 4\% per annum. For non-tabular discounts, the discount rate for statutory accounting is rarely greater than that used for IRS discounting, since the discount rate must be disclosed in the notes to the financial statements and SSAP \#65, "Property and Casualty Contracts," $T 12$, requires a 150 basis point margin.

12 A company with negative reserves (from anticipated salvage and subrogation or expected reinsurance recoverables) in an accident year would also face the limitation, since discounting the reserve makes it less negative and raises its dollar value.
${ }^{13}$ In practice, discount rates vary by characteristics of the bond, so this valuation method is not exact.
14 IFRS 4 [IFRS 17] uses a risk margin that reflects the insurer's risk aversion and the uncertainty of losses and allows insurers to adjust the discount rate for liquidity of the losses. The risk margin and illiquidity premium do not reflect the mainstream financial perspective on fair value.

15 The IASB and the FASB say that fair values are exit prices, determined at the measurement date, not entry prices, determined at the purchase or incurral date.

16 In contrast, book values for fixed assets in traditional GAAP do not change when interest rates change.
${ }^{17}$ The bond type depends on a contractual cash flow test (whether the cash flows consist of principal and interest only) and a business model assessment (whether the bond is held to collect contractual cash flows or whether it might be sold).

18 Traded assets have market values. Loss reserves have fair values, not market values, but the fair values estimate what the market value would be if a market existed. Economic value means market value if it exists or fair value if no market value exists.

19 Average interest rates over several years were once used to smooth temporary fluctuations. Most financial economists believe that these averages are poorer predictors of future rates.
${ }^{20}$ The average loss date in an accident year is the middle of the year. For policies with annual terms, the premium writing date is generally half a year before the average loss date, so the rate an the beginning of the accident year reflects the implicit interest rate on bonds bought with the policy premium.
${ }^{21}$ See section 846(c)(2) of the Internal Revenue Code: "Determination of Annual Rate: The annual rate determined ... for any calendar year shall be a rate equal to the average of the applicable Federal mid-term rates (as defined in section 1274(d) but based on annual compounding) effective as of the beginning of each of the calendar months in the test period. The test period is the most recent 60 -calendar-month period ending before the beginning of the calendar year for which the determination is made."

The federal mid-term rates are bond equivalent yields, since bond coupons are paid semi-annually in the United States. (A bond equivalent yield is a yield with semi-annual compounding.) The IRS loss reserve discounting procedure uses annual compounding, since it assumes that losses are paid in mid-year (i.e., once a year). The bond equivalent yields are converted to effective annual yields before averaging, using the formula $r_{a}=\left(1+r_{s} / 2\right)^{2}-1$, where $r_{a}$ is the effective annual yield and $r_{s}$ is the bond equivalent yield with semiannual compounding. If the bond yield is $8 \%$ per annum, the effective annual rate is $(1+0.08 / 2)^{2}-1=8.16 \%$.

The Treasury now promulgates federal mid-term rates with four types of compounding: annual, semi-annual, quarterly, and monthly. At December 1, 2016, these rates were: annual = 1.47\%; semi-annual, quarterly, and monthly $=1.46 \%$.
${ }^{22}$ The yield among mid-term securities varies with the remaining maturity, in accordance with the term structure of interest rates. More recently issued securities tend to have slightly lower yields, since they have higher demand. The Secretary of the Treasury selects an appropriate average rate.
${ }^{23}$ The difference between IRS discounted losses and IFRS 4 discounted losses also depends on the risk margin and the illiquidity premium in the interest rates, the duration of the losses, Treasury bond rates vs. other measures of the risk-free rate, and the actuarial loss payment patterns vs the IRS patterns.
${ }^{24}$ Combined factors are used for the Schedule P Lines of Homeowners-Farmowners, Commercial MultiPeril, and Special Liability.

25 This slows down the payment schedule somewhat, reducing the discounted loss reserves and the offset to taxable income, and raising taxable income and the tax liability. The error is not large, and it is offset by other characteristics of the IRS procedure which speed up the payment schedule. On the whole, the IRS loss reserve discount factors are not materially biased.
${ }^{26}$ Further below we show the standard actuarial method of estimating loss payment patterns; here we focus on the IRS method to derive the loss reserve discount factors.

27 The percentage of 2009 losses paid in 2010 can be derived from the 2010 Schedule $P$, which is not yet available at year-end 2010.

28 If the projected percentages of reserves paid in future years do not sum to $100 \%$, the percentages are normalized. If claims settled more quickly in the most recent calendar year, the total of the percentages is more than $100 \%$; if claims settled less quickly in the most recent calendar year, the total is less than $100 \%$.

29 Schedule P, Part 1, includes all loss adjustment expenses; Part 3 has only defense and cost containment. See section 846(f)(2) of the Internal Revenue Code: The term "unpaid losses" includes any unpaid loss adjustment expenses shown on the annual statement. In addition, Part 1 is audited; Part 3 is not audited.

30 For actuarial discount factors, one would use estimated ultimate losses.
${ }^{31}$ The figures in row 4 sum to $89.1 \%$. This is the ratio of cumulative paid losses to incurred losses for accident year 20X0, the oldest accident year shown in Part 1.

32 The $n^{\text {th }}$ year here means the $n^{\text {th }}$ year working backwards from the current valuation date. We estimate the amounts to be paid in future calendar years by looking at old accident years. The difference in the cumulative percentages paid between the $n^{\text {th }}$ and $(n+1)^{\text {rst }}$ past Schedule $P$ accident years is the percentage assumed to be paid between the end of the $\mathrm{n}^{\text {th }}$ and the $(\mathrm{n}+1)^{\text {rst }}$ calendar years from inception of the accident year to which the factors apply. The $\mathrm{n}^{\text {th }}$ accident year working backwards from the most recent Schedule P accident year corresponds to the $\mathrm{n}^{\text {th }}$ calendar year working forwards from the current tax year.

33 The assumption that all losses are paid at mid-year is a proxy for an even distribution of loss payments during the year. In fact, losses are incurred (on average) at mid-year, but claim reporting lags and the time needed to investigate and set up files cause the average payment date to be later than the middle of the year, particularly for the first year or two following the accident year.
${ }^{34}$ See the Internal Revenue Code $\S \S 846(d)(3)(C)$ and (D), "Special rule for certain long-tail lines": In the case of any long-tail line of business, the period taken into account shall be extended (but not by more than 5 years), and the amount of losses which would have been treated as paid in the 10th year after the accident year shall be treated as paid in such 10th year and each subsequent year in an amount equal to the amount of the losses treated as paid in the 9th year after the accident year (or, if lesser, the portion of the unpaid losses not theretofore taken into account). To the extent such unpaid losses have not been treated as paid before the last year of the extension, they shall be treated as paid in such last year. The term "long-tail line of business" means any line of business if the amount of losses which would be treated as paid in the 10th year after the accident year exceeds the losses treated as paid in the 9th year after the accident year.

35 See McClenahan [1975], Butsic [1981], and Sherman [19**].
${ }^{36}$ The formulas below assume two claims types (slow and fast-paying). In practice, claims fall along a range of very fast paying to very slow paying, with the same effect on the loss payment pattern.
${ }^{37}$ Negative loss payments can result from unanticipated salvage and subrogation (or a failure to accrue anticipated salvage and subrogation) or from unanticipated reinsurance recoverables (or a failure to accrue anticipated reinsurance recoverable). Loss payments are net of anticipated reinsurance and anticipated salvage and subrogation, even if the recoverable has not yet been received. Net paid losses in Schedule $P$ means direct plus assumed paid losses minus ceded paid losses. This means the direct plus assumed losses
paid minus the reinsurance recoverables received or expected on these loss payments. See SSAP No. 53, "Property-Casualty Contracts - Premiums," and SSAP No. 62, "Property and Casualty Reinsurance," not the direct plus assumed losses paid minus the reinsurance recoverables actually received. If the insurer does not anticipate a recoverable but receives unexpected reinsurance or salvage and subrogation in a later year, the recovery is a negative paid loss.
${ }^{38}$ See the Internal Revenue Code §846(d)(3)(G): "If the amount of the losses treated as paid in the 9th year after the accident year is zero or a negative amount, subparagraphs (C)(ii) and (D) shall be applied by substituting the average of the losses treated as paid in the 7th, 8th, and 9th years after the accident year for the losses treated as paid in the 9th year after the accident year." The cap for the years subsequent to the tenth year is changed, but the negative assumed payment for the tenth year remains.
${ }^{39}$ If the average of the three oldest accident years is still negative, the average of the four oldest years is used; if this average is still negative, the average of the five oldest years is used. One continues in this fashion until one comes to the average of ten years, which must be positive.
${ }^{40}$ For industry data, the same loss payment pattern is used for five accident years; for company data, the loss payment pattern differs for each accident year. Different discount rates are used for each accident year regardless of industry vs company data.

41 Each discount factor is the ratio of discounted reserves to undiscounted reserves for a given accident year at a given valuation date. For instance, the accident year 2010 discount factor at the tenth maturity is the discounted reserves for accident year 2010 at 12/31/2020 divided by the undiscounted reserves for accident year 2010 at 12/31/2020. This discount factor is computed in 2010, not in 2020.
${ }^{42}$ Weighting the discount factors by the percentage of incurred losses by accident year in the prior years row is more accurate and was permitted before 1988, but the distribution of the prior years row reserves by accident year is not shown. The composite discount factor approach was introduced in 1988 as an alternative to accident year weights. See Treasury Notice 88-100, §V, which explains the computation of the composite discount factor shown here. The IRS aggregate discount factors shows two options for the prior years row: one factor for all prior years or five factors, with the factor for the oldest year also used for preceding years.
${ }^{43}$ See IRC §846(e)(2)(C): "An election ... with respect to any determination year shall be made on the taxpayer's return for the taxable year in which ... the determination year ends."

44 See IRC $\S 846(e)(2)(B)$ : "an election ... shall apply to accident years ending with the determination year and to each of the 4 succeeding accident years."

45 If the insurer uses its own data, the payment pattern for accident year 20XX is computed from the Schedule P for 20XX-2, which is the last Schedule P filed before the beginning of accident year 20XX.
${ }^{46}$ See IRC §846(e)(3): "No election ... shall apply to any international or reinsurance line of business"; see also §846(d)(3)(E).
${ }^{47}$ See IRC §846(e)(4)(B): "an election under this subsection applies to all lines of business of the taxpayer," and 2001FED $26,330 \mathrm{C}, \S 1.846-2$, Election by taxpayer to use its own historical loss payment pattern: "A taxpayer making the election must use its own historical loss payment pattern in discounting unpaid losses for each line of business that is an eligible line of business in that determination year."
${ }^{48}$ Reserving actuaries should keep work-papers showing reserve indications with risk margin or reserve margin. An actuary who believes the insurer might book a higher reserve would do well to provide a range of indications, so that the IRS does not assume the insurer has added a reserve margin.

49 See Regulation 88-100, §III: "Until further guidance is issued, such statistically significant amount is business in at least the $10^{\text {th }}$ percentile of industry-wide reserves for a line of business for the determination year with respect to which the election is made."
${ }^{50}$ See Treasury regulation 2001FED $26,330 \mathrm{C}, \S 1.846$-2, Election by taxpayer to use its own historical loss payment pattern: "A line of business is an eligible line of business in a determination year if ... the taxpayer reports losses and loss expenses incurred ... for at least the number of accident years for which losses and loss expenses incurred for that line of business are ... separately reported on that annual statement."
${ }^{51}$ See IRC §846(e): "An insurance company is required to take estimated salvage recoverable (including that which cannot be treated as an asset for state statutory accounting purposes) into account in computing the deduction for losses incurred."

52 See the Internal Revenue Code, section 846(2) "A company is allowed to increase the unpaid losses shown on its annual statement only if the company . . . discloses on its annual statement, by line of business and accident year, the extent to which estimated salvage recoverable is taken into account in computing the unpaid losses shown on the annual statement . . ."

Until 2001, insurers could use their own discount factors for anticipated salvage and subrogation. See Treasury regulation 2001FED $26,153, \$ 1.832-4$, says that "except as otherwise provided in guidance published by the Commissioner in the Internal Revenue Bulletin, estimated salvage recoverable must be discounted either (1) by using the applicable discount factors published by the Commissioner for estimated salvage recoverable; or (2) by using the loss payment pattern for a line of business as the salvage recovery pattern for that line of business and by using the applicable interest rate for calculating unpaid losses under section 846(c)." The option to use one's own discount factors was revoked in 2001.

54 See IRS Rev. Proc. 2002-74 §2, ๆ.03: "losses incurred during the taxable year ... [are] (1) from losses paid during the taxable year, deduct salvage and subrogation recovered; (2) add ... all discounted unpaid losses outstanding at the end of the taxable year and deduct all discounted unpaid losses outstanding at the end of the preceding taxable year; (3) add estimated salvage and reinsurance recoverable as of the end of the preceding taxable year and deduct estimated salvage and reinsurance recoverable as of the end of the taxable year. The amount of the estimated salvage recoverable is determined on a discounted basis."

