

**Statutory Surplus:
Computation, Pricing, and Valuation**

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Casualty Actuarial Society Exam 7 Study Note

Statutory Surplus: Computation, Pricing, and Valuation

The development of financial pricing models for insurance products and the advent of risk-based capital requirements have led to an increasing focus on capital. This study note explains the differences between statutory surplus and invested capital and the implications for actuarial pricing and valuations.

When insurance products were priced to a 5% underwriting profit margin, some states mandated that investment income be considered, often by simple *investment income offset* procedures that reduced the underwriting profit margin for the investment income earned on policyholder supplied funds.¹ These procedures did not consider capital or surplus.

Insurers are now using *return on capital* models, for pricing and performance measurement. The pricing models vary in their definitions of *capital*, whether statutory surplus, GAAP equity, economic surplus, or invested capital. Similarly, actuarial valuations may use GAAP earnings, statutory earnings, or cash flows. The valuation results differ sharply, and proper valuation relies on an accurate assessment of the capital supporting insurance operations.

The idiosyncracies of statutory accounting complicate the relation between statutory surplus and invested capital. For example, the statutory income statement shows *net income* earned during the current year. One might suppose that last year's surplus plus this year's net income should equal this year's surplus, but there are half a dozen other adjustments, such as the *change in non-admitted assets* or the *change in the provision for reinsurance*, which are *direct charges or credits* to surplus. In contrast, the surplus figure on the balance sheet is total assets minus total liabilities, with no adjustment for *direct charges or credits to surplus*. The sign of the income statement adjustments is also confusing, since an increase in non-admitted assets causes a decrease in policyholders' surplus. Some readers wonder: "Why should an increase in assets, of whatever sort, lead to a decrease in the worth of the company?"

This study note traces the computation of statutory surplus, along with the difference from GAAP equity; contrasts statutory surplus (and GAAP equity) with invested capital; and adds the capital in the policyholder reserves with the capital required by statutory regulations to determine the capital supporting the insurance policy.²

BALANCE SHEETS AND INCOME STATEMENTS

Surplus has two definitions. The balance sheet definition says *surplus = assets – liabilities*. The income statement definition says *surplus = last year's surplus + current year's income*.

If all balance sheet transactions also flowed through the income statement, and if all income

statement transactions had corresponding effects on statutory assets and liabilities, the two definitions of surplus would be equivalent, and no adjustments would be needed.

Illustration: Suppose an insurer begins the year with \$2,000 in surplus and \$2,000 in cash, and it writes a policy for a \$1,000 premium on January 1. An insurer normally hold marketable securities, not cash; we use cash in this illustration to avoid the accounting entries for investment income. The insurer incurs expenses of \$250 and losses of \$600 during the year.

Statutory accounting is on an accrual basis, not a cash basis. It makes no difference whether the premium has been collected or is still owed the company (so long as the receivable is admitted) and no difference whether the losses are paid or held as reserves. Let us suppose the premium is collected on January 1, expenses are paid during the year, \$200 of losses are paid by December 31, and \$400 of losses remain in reserves.

During the year, the cash account increases by the \$1,000 premium and decreases by the \$250 of expenses and the \$200 of paid losses, for a net increase of \$550. At the end of the year, the cash account has the original \$2,000 plus the year's increase of \$550, for a total of \$2,550. Liabilities, which were \$0 at the beginning of the year, increase by the \$400 case reserves. Surplus at year end is \$2,150, or the \$2,550 of assets minus the \$400 of liabilities.

From an income statement perspective, premium earned is a revenue, and losses incurred and underwriting expenses are expenditures.³ Net income = revenues minus expenditures = \$1,000 – \$600 – \$250 = \$150 is the addition to surplus during the year.

At the year end, income statement surplus has increased by \$150 (revenues – expenditures). Balance sheet surplus of \$2,150 equals total assets (\$2,550) minus total liabilities (\$400).

<i>Activity</i>	<i>Accounting Entry</i>	<i>Account</i>	<i>Debit</i>	<i>Credit</i>
	Cash	Asset	\$2,000	
	Policyholders'	Surplus		\$2,000
Write Policy	Cash	Asset	\$1,000	
	Unearned Premium	Liability		\$1,000
Incur Expenses	Expenses incurred	Expenditure	\$250	
	Cash	Asset		\$250
Incur Losses	Losses Incurred	Expenditure	\$600	
	Cash	Asset		\$200
	Loss Reserve	Liability		\$400
Non-Ledger	Unearned Premium	Liability	\$1,000	
	Premiums Earned	Revenue		\$1,000

In the accounting presentation above, ledger transactions are entered onto an accounting ledger; year end adjustments are non-ledger items.

The balance sheet and income statement definitions of surplus are not the same if some balance sheet transactions do not flow through the income statement. Nonadmitted assets and statutory liabilities affect the balance sheet; the income statement does not differentiate between admitted and non-admitted assets, it is not affected by statutory liabilities.

NON-ADMITTED ASSETS

If \$100 of premium remains uncollected and more than 90 days past due on December 31, it is not admitted. The income statement entries remain earned premium (\$1,000), incurred losses (\$600), and expenses (\$250), for a net income of \$150. The income statement surplus, before any adjustments, would be last year's surplus of \$2,000 plus the net income of \$150.

The balance sheet recognizes only the admitted portion of assets. The premium collected of \$900 increases cash by \$900. The remaining \$100 of earned premium appears as *premiums and agents' balances in course of collection* (page 2, line 10.1). Since it is overdue, it appears in column 2, *Assets Not Admitted*, and it does not enter into balance sheet surplus. The balance sheet calculation of policyholders' surplus is as follows:

At the beginning of the year, *cash on hand or on deposit* is \$2,000. We add to the cash account the collected premium of \$900 and subtract expenses paid of \$250 and losses paid of \$200 to *get cash on hand or on deposit* of \$2,450 at the end of the year. The \$100 of uncollected premium more than 90 days past due is not admitted and does not appear in the *net admitted assets* column of the statutory balance sheet. The liability side of the balance sheet shows the case reserve of \$400 and surplus of $\$2,450 - \$400 = \$2,050$.

THE ASSET EXHIBIT

To reconcile income statement surplus with balance sheet surplus, we adjust income statement surplus for transactions and statutory accounts that do not *flow through the income statement*. Exhibit 1, "Analysis of Non-Admitted Assets and Related Items" (page 13 of the Annual Statement) shows the *change in nonadmitted assets during the year*, which is the needed adjustment to the income statement surplus.

Why do we want the *change* in non-admitted assets instead of the non-admitted asset itself? And why does an *increase* in non-admitted assets lead to a *decrease* in surplus?

Consider again the illustration above. Earned premium during the year is \$1,000. But the \$1200 of premium receivable at year end is not admitted, since it is more than 90 days past due. The *increase* in non-admitted assets during the year, from \$0 to \$100, really means a

decrease in the admitted portion of the assets.

Do not think of this as a fixed admitted asset to which is tacked on a non-admitted asset. Rather, conceive of the *total* asset as a *fixed* amount, so an *increase* in the non-admitted portion is a *decrease* in the admitted portion. The earned premium on the income statement is an increase in total assets. If total assets increase during the statement year by \$1,000, and non-admitted assets increase by \$100, then admitted assets increase by only \$900.

The income statement shows revenues, which correspond to the increase in *total* assets. *Subtracting* the increase in non-admitted assets gives the increase in admitted assets. The increase in non-admitted assets is a direct charge to policyholders' surplus.

THE STATUTORY BALANCE SHEET

The statutory balance sheet uses four columns to reconcile with the income statement.

- Column 1: (Total) assets
- Column 2: Assets not admitted
- Column 3: Net admitted assets (Columns 1 – 2)
- Column 4: Net admitted assets (prior year)

Agents' balances are in column 1. The portion more than 90 days past due is recorded in column 2 and the difference is the net admitted asset in column 3. The change in the non-admitted asset appears in Exhibit 1, Analysis of Non-Admitted Assets and Related Items:

- Column 1: Non-admitted assets at the end of the current year
- Column 2: Non-admitted assets at the end of the previous year
- Column 3: Change for year (increase) or decrease, or column 2 – Column 1.

A positive entry in column 3 means a *decrease* in non-admitted assets, and a negative entry in column 3 means an *increase* in non-admitted assets.

SURPLUS ADJUSTMENTS

An increase in non-admitted assets, given fixed total assets, is a decrease in net admitted assets; the negative entry in column 3 of Exhibit 1 is carried to page 4 and reduces surplus. A decrease in non-admitted assets is an increase in net admitted assets; the positive figure in column 3 of Exhibit 1 increases surplus on page 4.

The non-admitted assets in Exhibit 1 include only the assets on lines 10-17 and 19-21 of the balance sheet plus certain other assets: (i) bills receivable, past due, taken for premium, (ii) furniture and equipment, and (iii) loans on personal security. Exhibit 1 does not include the non-admitted portions of financial assets (lines 1-9 of the balance sheet), or the *excess of*

book over market (or amortized) values. The *change* in the excess of book over market from one year to the next is the unrealized capital gain or loss. The *net unrealized capital gains or losses* are shown as a separate adjustment to surplus page 4.

OFFICE FURNITURE

We show several examples of entries peculiar to statutory accounting. Insurers have two statutory accounting options for non-admitted assets.

- Method 1: Write off the non-admitted asset as an expense in the income statement.
- Method 2: Use GAAP entries for the balance sheet and the income statement, but classify the asset as non-admitted with a direct charge to surplus.

Suppose an insurer buys office furniture on December 31, 20X4, with a useful life of 10 years for \$100,000; the insurer uses straight line depreciation. The 20X4 GAAP entries are:

- Credit cash by \$100,000 (cash paid to purchase furniture).
- Debit an office furniture asset by \$100,000.

Both entries are on the balance sheet, and there is no effect on GAAP equity. These are ledger entries; the purchase of the furniture is shown on the accountant's ledger.

For statutory financial statements, the Method 1 accounting entries are

- Credit cash by \$100,000 (cash paid to purchase furniture).
- Debit general expenses (income statement) by \$100,000.

The entries are on different financial statements, and statutory surplus declines by \$100,000.

The Method 2 accounting transactions are

- Credit cash by \$100,000 (cash paid to purchase furniture).
- Debit an office furniture asset by \$100,000.
- Enter \$100,000 in the non-admitted column for the office furniture asset.
- The non-admitted assets increase from \$0 before the purchase of the furniture to \$100,000 after the purchase of the furniture. The change in non-admitted assets of +\$100,000 is a direct charge to surplus.

The year-end 20X5 GAAP non-ledger entries are

- Credit the office furniture asset by \$10,000 to reflect depreciation.
- Debit depreciation expense (income statement) by \$10,000.

For statutory accounting, if Method 1 is used for the initial purchase, there are no accounting transactions in subsequent years; the full \$100,000 was an expense in 20X4. The Method 2 accounting transactions are

- Credit the office furniture asset by \$10,000 to reflect depreciation.
- Debit depreciation expense by \$10,000.

The non-admitted office furniture declines from \$100,000 to \$90,000. The -\$10,000 change in non-admitted assets is a credit to surplus, offsetting the debit from the income statement.

GAAP depreciates the office furniture by \$10,000 each year to match revenue and expenses. Statutory Method 1 says that the office furniture has little or no realizable value. It can not be used to pay claims, so its entire value is written off when it is purchased.

Method 1 requires two sets of books: one for GAAP and one for statutory accounting. This complicates the accounting, and it may lead to errors. Method 2 uses GAAP books only, but it non-admits certain assets. The income statement entries are the same as for GAAP statements; any changes needed are made by direct charges and credits to surplus.

ACCRUED RETROSPECTIVE PREMIUMS

Accrued retrospective premiums are taken from the Underwriting and Investment Exhibit, "Recapitulation of All Premiums," page 8, Part 2A, column 5, line 33, "accrued retrospective premiums based on experience," and entered on page 2, line 10.3, column 1. The non-admitted portion (usually 10% of the unsecured portion) is entered in column 2 and the difference is entered in column 3.

STATUTORY LIABILITIES: PROVISION FOR REINSURANCE

Any transaction that affects the balance sheet but not the income statement is a direct charge or credit to surplus. For instance, an increase in the Schedule F provision for reinsurance does not flow through the income statement but it increases liabilities on the balance sheet, thereby decreasing balance sheet surplus. The increase (decrease) in the provision for reinsurance from the previous year to the current year is a direct charge (credit) to surplus.

Illustration: Suppose an insurer has a 50% pro-rata reinsurance treaty with an authorized reinsurer. A loss occurs on March 1 and a direct case reserve of \$200,000 is posted. On June 1, the loss is paid for \$300,000. At year end, the reinsurance recovery has not been collected and it is more than 90 days past due. The financial statement entries are

March 1: Debit incurred losses \$200,000 (direct loss, income statement)
Credit incurred losses \$100,000 (reinsurance recoverable, income statement)
Credit case reserve \$200,000 (direct loss, balance sheet)

Debit case reserve \$100,000 (reinsurance recoverable, balance sheet)

June 1: Debit incurred losses \$100,000 (direct loss, income statement)
Credit incurred losses \$50,000 (reinsurance recoverable, income statement)
Debit case reserve \$200,000 (direct loss, balance sheet)
Credit case reserve \$100,000 (reinsurance recoverable, balance sheet)
Credit cash \$300,000 (direct loss, balance sheet)
Debit reinsurance recoverable \$150,000 (balance sheet)

Dec 31: Credit provision for reinsurance \$30,000 (balance sheet)
Change in provision for reinsurance \$30,000 (direct charge to surplus)

UNREALIZED CAPITAL GAINS

Unrealized capital gains are direct credits to surplus. Suppose that on December 31, 20X4, an insurer has \$100 million of assets, \$60 million of liabilities, and surplus of \$40 million. The assets are 80% bonds and 20% common stock. In 20X5, the stocks increase in value to \$30 million. The federal income tax rate is 35%. The 20X5 financial statement entries are

- Debit stocks \$10 million (balance sheet)
- Credit deferred tax liability \$3.5 million (balance sheet)
- Unrealized capital gains of \$10 million (direct credit to surplus)
- Change in deferred tax liability of \$3.5 million (direct charge to surplus)

AUDIT PREMIUMS

We show the accounting entries for a \$10,000 policy written on October 1, 20X3, with an estimated audit premium of \$2,000. The estimated earned premium for the full policy term is \$12,000, of which the 20X3 portion is \$3,000. Estimates of audit premiums may be included as written premium or as a separate adjustment to earned premium. The accounting entries on 12/31/20X3 are either

- written premium of \$12,000 and an UEPR of \$9,000 or
- written premium of \$10,000 and an UEPR of \$7,000.

DEFERRED POLICY ACQUISITION COST AND PREMIUM DEFICIENCY RESERVE

Suppose an insurer writes a block of policies with written premium of \$100 on July 1, 20X4. Acquisition costs are \$20 million and expected losses are \$80 million; investment income covers other expenses. GAAP recognizes the premium and the expenses over the term of the policy by setting up both an unearned premium reserve and a DPAC (deferred policy acquisition cost) asset and amortizing them over the policy term. On December 31, the remaining UEPR is \$50 million and the remaining DPAC is \$10 million, for a net reserve of

\$40 million. We show the GAAP and statutory accounting entries for two scenarios:

If by December 31, 20X4, incurred losses are \$45 million, and the insurer expects another \$45 million of incurred losses in the next six months, the DPAC is reduced to \$5 million, and expenses are debited by \$5 million on the income statement. Statutory has no DPAC, so no accounting entries are needed

If by December 31, 20X4, incurred losses are \$65 million, and the insurer expects another \$45 million of incurred losses in the next six months, the DPAC is reduced to zero, and a premium deficiency reserve of \$15 million is set up on both GAAP and statutory statements.

INTEREST DUE AND ACCRUED

Suppose an insurer buys \$100 million of investment grade 6% coupon bonds on March 1, 20X4, and classifies them as available for sale (FAS 115). By December 31, 20X4, interest rates have declined and the market value of the bonds is \$102 million. In 20X5, the issuer fails to pay the August 31 coupon, and the bonds are downgraded to class 4. On December 31, 20X5, the market value of the bonds is \$90 million; the August 31 coupon is still not paid, but the company expects to collect it next month. We show the accounting entries.

20X4: The cash received of \$3 million and the accrued interest of \$2 million are revenues (credits) on the income statement and debits to cash and to interest receivable on the balance sheet. On the GAAP balance sheet, the bonds are marked to market (\$102 million). The \$2 million increase is a direct credit to equity; it does not flow through the income statement. On the statutory balance sheet, the bonds remain at (amortized) cost of \$100 million.

20X5: The cash received on February 28 of \$3 million and the accrued interest of -\$2 million are revenues on the income statement and debits to cash and interest receivable on the balance sheet. By year-end, the bond has been downgraded to Class 4, and it is shown at market value on both GAAP and statutory financial statements. GAAP shows a \$12 million charge to equity, and statutory accounting shows a \$10 million charge to surplus.

GAAP shows \$3 million as interest receivable and \$2 million as interest due and accrued; the full \$5 million flows through the income statement. Statutory accounting does not admit any of the interest, since the payments is more than 90 days past due. Method 1 shows no balance sheet or income statement entries. Method 2 shows the same entries as GAAP and then classifies the assets as non-admitted and have a \$5 million direct charge to surplus.

REAL ESTATE

On December 31, 20X4, an insurer buys a shopping mall for \$50 million as a real estate investment. Rental income is \$8 million a year, and depreciation is \$2 million a year for 25 years. On December 31, 20X5, the market value of the mall has increased to \$53 million. In

20X6, a competing shopping mall opens 4 miles away, and by December 31, 20X6, the market value of the insurer's shopping mall is \$43 million.

- 2004: Cash is credited \$50 million and investment real estate is debited \$50 million; there is no change in surplus.
- 2005: Rental income flows through the income statement at investment income (\$8 million credit) and cash is debited \$8 million. Depreciation expense is debited \$2 million (income statement), and investment real estate is credited \$2 million. No entry is made for the increase in market value.
- 2006: The rental income and depreciation entries are the same as for 2005. The book value of the real estate is \$46 million, of which \$3 million is not admitted (excess of book over market value), and there is a \$3 million direct charge to surplus.

STOCKHOLDER DIVIDENDS AND CAPITAL CONTRIBUTIONS

An insurer begins the year with \$100 of 8% coupon bonds maturing on December 31 in five years. The tax rate is 35%, and taxes are paid when cash is received. The insurer remits the after-tax investment income to its shareholders. On December 31, the insurer sells an additional one million shares of common stock, with a par value of \$1 per share and a sale price of \$1.50 per share. We show the accounting entries.

On June 30, the insurer receives \$4 million of bond interest: $\$4 \text{ million} \times 35\% = \1.4 million is paid to the Treasury and \$2.6 million are shareholder dividends; the same transactions occur on December 31. The accounting entries on each date are

- Debit cash \$4 million (balance sheet)
- Credit investment income \$4 million (income statement)
- Credit cash \$1.4 million (balance sheet)
- Debit tax liability \$1.4 million (income statement)
- Credit cash \$2.6 million (balance sheet)
- Shareholder dividend \$2.6 million (direct charge to surplus)

The accounting entries for the common stock issue are

- Debit cash \$1.5 million (balance sheet)
- Credit common capital stock \$1 million for par value of common stock (balance sheet)
- Credit paid-in and contributed surplus \$0.5 million for excess of sale price over par value (balance sheet)
- Direct credits to surplus: \$1 million for capital paid in and \$0.5 million for surplus paid in.

STATUTORY SURPLUS, GAAP EQUITY, AND CAPITAL INVESTED

A misconception that is sometimes heard in actuarial circles runs as follows: For statutory accounting purposes, we must understand the computation of statutory surplus. In some states, we might need surplus amounts for rate filings as well. For actuarial pricing of insurance products, however, we may dispense with statutory numbers. We seek a return on the economic capital needed to support the insurance operations. We determine this capital by actuarial techniques such as probabilities of ruin or expected policyholder deficits.

Many years ago, actuaries priced products to achieve a pre-set underwriting profit margin, such as 5% for most lines or 2.5% for workers' compensation. This pricing technique did not allow a comparison of insurance profitability with profitability in other industries. An early attempt by Arthur D. Little to examine insurance profitability looked at the return on assets (ROA), or the income during the year divided by the assets held by the insurance company.

The return on assets supposedly shows how efficiently insurers are using their assets to produce insurance policies, just as the ROA for an auto manufacturer shows how efficiently it uses its assets to produce automobiles. But insurers do not use their assets to produce insurance policies. An insurer might invest its money in the bonds issued by an automobile manufacturer; the assets represented by these bonds are used to manufacture automobiles, not automobile insurance policies.

Some financial analysts apply return on equity measures to insurance, looking at the ratio of GAAP income to GAAP equity. Ferrari [1967] examines the calendar year profitability of the insurance industry. Pricing actuaries, concerned with prospective ratemaking, look at *benchmark equity* or *benchmark surplus*: the equity or surplus needed to support the insurance operations, not the equity or surplus currently held by the company or by the industry.

In other industries, the return on equity is a proxy for the return on invested capital. For property-casualty insurance, GAAP equity is not the same as invested capital. Invested capital is statutory surplus plus the capital embedded in gross unearned premium reserve and full value loss reserves. The invested capital implied by statutory surplus is the crux of financial pricing and valuation.

DOUBLE TAXATION

The valuation of an insurance company requires an adjustment for the cost of holding capital. The cost of holding capital is at least the cost of double taxation (Myers and Cohn [1987]) and perhaps as high as the difference between the cost of equity capital and the after-tax investment yield (Atkinson and Dallas [2000]).⁴

Suppose investors must contribute \$100 million to support the writing of insurance policies, and this capital is invested in 10% coupon taxable bonds. If they invest the capital themselves,

the investors pay personal income taxes on the \$10 million return. If the insurer makes the same investment, it pays \$3.5 million of corporate income taxes and remits the remaining investment income to the investors, who pay personal income taxes on this dividend. The cost of double taxation is the difference in the taxes incurred between direct and indirect investment of capital.⁵

- The taxes paid on direct investment of capital = $investment\ yield \times personal\ tax\ rate$.
- The taxes paid on investment of capital through an insurance company = $investment\ yield \times [corporate\ tax\ rate + (1 - corporate\ tax\ rate) \times personal\ tax\ rate]$
- The difference between these two is $investment\ yield \times [corporate\ tax\ rate + (1 - corporate\ tax\ rate) \times personal\ tax\ rate - personal\ tax\ rate]$
 $= investment\ yield \times corporate\ tax\ rate \times (1 - personal\ tax\ rate)$

If the investment yield is 10%, the corporate tax rate is 35%, and the average personal tax rate is 30%, the cost of holding capital is $10\% \times [35\% + (1 - 35\%) \times 30\% - 30\%] = 10\% \times 35\% \times (1 - 30\%) = 2.45\%$. The investors pay an additional 2.45% of the yield on their capital to the taxing authorities. This is the *after-tax* loss to the investors. The loss before personal income taxes is the investment yield \times the corporate tax rate or $10\% \times 35\% = 3.5\%$. To induce investors to fund the insurance operations, the 3.5% of lost yield must be paid by the policyholders, not the investors.⁶

If the policyholders paid this money directly to the investors, this would be the full cost of holding capital. But there are no direct transactions between policyholders and investors. The policyholders pay this money as part of the policy premium, and the insurer remits the money to the investors. This introduces another layer of tax, since the policy premium is pre-tax and the compensation to the investors is post-tax. The needed margin in the policy premium, as a percentage of the investment yield on investor supplied capital, is

$$investment\ yield \times corporate\ tax\ rate / (1 - corporate\ tax\ rate) = investment\ yield \times 35\% / (1 - 35\%) = investment\ yield \times 53.85\%^7$$

The double taxation affects invested capital, whereas the money paid by policyholders is a margin on premium. The needed margin is $capital \times investment\ yield \times 53.85\% / premium$. If the premium is paid at policy inception and the taxes are paid (on average) at mid-year, the needed margin is $capital \times investment\ yield \times 53.85\% / [premium \times (1 + investment\ yield)^{1/2}]$.

Atkinson and Dallas [2000] define the cost of holding capital as the difference between the cost of equity capital and the after-tax investment yield of the insurance company. To illustrate, suppose the cost of equity capital is 12% per annum, but the insurance enterprise invests in 8% Treasury securities. The cost of double taxation is $35\% \times 8\% = 2.8\%$. The additional cost stemming from the conservative investments of the insurance company is $12\% - 8\% = 4\%$, and the total cost of holding capital is $2.8\% + 4\% = 6.8\%$. This is the amount that policyholders must pay to the investors to induce them to fund the insurance operations. Since the

policyholders pay this money indirectly through the profit margin in the premium, which is taxed as underwriting income, the additional premium is $6.8\% / (1 - 35\%) = 10.46\%$. If the premium is paid at policy inception and the taxes are paid (on average) at mid-year, the profit margin is $10.46\% / 1.08^{1/2} = 10.07\%$.

This implies that with an 8% investment yield and a 400 basis point spread between the target return on capital and the investment yield, the policyholders pay 10% of investor supplied capital to compensate for the indirect investment of their funds. The needed underwriting profit margin to be combined with expenses and discounted losses is 10% divided by the premium to capital ratio. If discounted losses and fixed expenses are \$2,800, the variable expense ratio is 22%, and the premium to capital ratio is 1.25, the needed underwriting profit margin is $10\% / 1.2 = 8\%$, and the indicated premium is $\$2,800 / (1 - 22\% - 8\%) = \$4,000$.

The cost of holding capital depends on both the capital explicitly held as surplus and the capital embedded in statutory reserves. We illustrate the cost of double taxation both with and without consideration of deferred tax assets:

An insurer operates at a two to one premium to surplus ratio. Each year, written premium is \$200 million, the unearned premium reserve is \$100 million, the pre-paid acquisition expense ratio is 25% of written premium, and the undiscounted loss reserves are \$300 million. The risk-free interest rate is 5% per annum and the company's investment yield is 8% per annum. The IRS loss reserve discount factor is 80% for all years and all valuation dates (and the tax basis reserves are at fair value), and 25% of held reserves are paid out during the next year.

If deferred tax assets are not considered, the invested capital is \$100 million of surplus + 25% × \$100 million = \$25 million of equity in the unearned premium reserve + $(1 - 80\%) \times \$300$ million = \$60 million in the undiscounted loss reserves, for a total of \$185 million. The cost of double taxation using the Myers' Theorem is $\$185 \text{ million} \times 5\% \times 35\% = \3.24 million. The cost of double taxation using the company's investment yield is $\$185 \text{ million} \times 8\% \times 35\% = \5.18 million. The Atkinson and Dallas cost of holding capital is $\$185 \text{ million} \times 6.8\% = \12.58 million.

The statutory deferred tax asset stemming from revenue offset is $35\% \times 20\% \times$ the unearned premium reserve = $35\% \times 20\% \times \$100 \text{ million} = \$7$ million. The statutory deferred tax asset stemming from loss reserve discounting is $35\% \times$ the reserve discount that is expected to reverse in the next 12 months, or $35\% \times 25\% \times 20\% \times \$300 \text{ million} = \$5.25$ million.

The invested capital is $\$185 \text{ million} - \$7 \text{ million} - \$5.25 \text{ million} = \172.75 million. The cost of double taxation using the Myers' Theorem is $\$172.75 \text{ million} \times 5\% \times 35\% = \3.02 million. The cost of double taxation using the investment yield of the company is $\$172.75 \text{ million} \times 8\% \times 35\% = \4.84 million. The Atkinson and Dallas cost of holding capital is $\$172.75 \text{ million} \times 6.8\% = \11.75 million.

Valuation: Cost of Holding Capital:

We continue the illustration to show the valuation of the company. The insurer has \$100 million of surplus of which \$12.25 million are deferred tax assets; \$200 million of written premium each year, \$100 million of unearned premium reserves, and \$300 million of undiscounted loss reserves. The cost of equity capital is 12% per annum, and the tax rate is 35%. The pre-paid acquisition expense ratio is 25% of written premium, and the discount factor for loss reserves is 80%. After-tax net income is remitted to shareholders.

The invested capital is \$100 million (surplus) + 25% × \$200 million (equity in UEPR) + 20% × \$300 (equity in undiscounted loss reserves) – \$12.25 (DTA) = \$172.75 million.

If the insurer expects to earn \$36 million of pre-tax income each year, the after-tax net income is \$36 million × (1 – 35%) = \$23.40 million. The present value of the future net income is \$23.4 million / 12% = \$195 million. The company is profitable; its net worth is \$195 million.

If the insurer expects to earn \$30 million of pre-tax income each year, the after-tax net income is \$30 million × (1 – 35%) = \$19.5 million. The present value of the future net income is \$19.5 million / 12% = \$162.50 million. The company is not profitable; the shareholders would gain by liquidating the company and taking the \$172.75 million. If the costs of liquidation are more than \$172.75 – \$162.50 = \$10.25 million, the company should continue operating.

* * * * *

Statutory surplus and GAAP equity of property-casualty insurance companies differ from invested capital. For pricing insurance products of valuing an insurance company, actuaries must be careful to include all capital in their analyses. This requires a complete understanding of statutory accounting, with particular emphasis on direct charges and credits to surplus.

Endnotes:

- ¹ See Robbin, "The Underwriting Profit Provision" [1992], algorithms 1 and 2.
- ² The capital invested in reserves is sometimes larger than the capital explicitly held as statutory surplus.
- ³ We use the term *expenditures*, to avoid confusion with underwriting expenses; accountants say *expenses*.
- ⁴ Cf the AAA Standard of Practice on Valuations. Sturgis [1981] takes the view of Atkinson and Dallas, but he leaves out the cost of double taxation; this is an inadvertent omission, not a difference of opinion. Miccolis, commenting on Sturgis, notes that Sturgis ignore risks; Miccolis [1987] follows Myers and Cohn, though he also omits the cost of double taxation.
- ⁵ The cost of double taxation may change with the 2003 tax amendments now before the Congress.

⁶ Myers asserts that the cost of double taxation is the same regardless of the investment portfolio of the insurer. If the cost of double taxation is \$20 million if the insurer holds Treasury securities, the cost of double taxation is \$20 million even if the insurer holds risky securities with a higher expected return and higher expected tax liabilities; see Derrig [1995]. According to Myers, just as the present value of the return from risky securities equals the present value of the return from risk-free securities, the present value of the federal income taxes on the investment income from risky securities equals the present value of the federal income taxes on the investment income from risk-free securities.

⁷ If one dollar of investment income is received directly, the IRS takes about 30¢. If one dollar of investment income is earned through an insurance company, the IRS takes 83.85¢.

